PRINCE OF SONGKLA UNIVERSITY
FACULTY OF ENGINEERING

Final Examination: Semester I — Academic Year: 2003

Date: September 30, 2003 — Time: 9:00-12:00

Subject: 230-601 Advanced Engineering Mathematics for Chemical Engineers — Room: R300

[illegible]

Please do all 5 questions. Show all your work to receive full or partial credit.
Total score is 160.

Question #	Total Score	Score
1	20	
2	20	
3	30	
4	40	
5	40	
Bonus	**10**	
Total	**160**	

[illegible]

[illegible] ([illegible]) [illegible]

1. Using Power Series solve the differential equation (20 points)

$$\frac{d^2y}{dx^2} - x\frac{dy}{dx} - x^2y = 0$$

2. Using Laplace Transform solve the differential equation (20 points)

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 4t \quad y(0) = 1,\ y'(0) = -1$$

3. Heat transfer from circular pipe to circular metal fins, temperature in circular metal fins express a heat transfer equation as following: (30 points)

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - x^2y = 0$$

Define: $y = T - T_A$ and $x = r\sqrt{\frac{2h}{bk}}$

If the pipe of radius R_p takes a temperature T_p and the outer rim of the fin at position R exists at temperature T_A, show that the temperature profile is

$$\frac{T - T_A}{T_P - T_A} = \frac{I_o(x)K_0(x_R) - K_0(x)I_0(x_R)}{I_0(x_P)K_0(x_R) - I_0(x_R)K_0(x_P)}$$

where $x_R = R\sqrt{\frac{2h}{bk}}$ and $x_P = R_P\sqrt{\frac{2h}{bk}}$

4. A semi-infinite rod with constant physical properties has an initial and boundary conditions as follow: (40 points)

$t = 0 \rightarrow T = To$

$x = 0 \rightarrow T = T_1$ and the other end is insulated

Use Newton's law and constant heat transfer coefficient (h) develop the partial differential equation describing the temperature of the semi-infinite rod as the function of time t and position x and **solve PDEs by using Laplace transform.**

Define: $\theta = \frac{T - T_0}{T_1 - T_0}$

5. Using **Separable Method of Variables** to find temperature profile of a slab if we know the transient of heat equation in the slab is, (40 points)

$$\frac{\partial T(x,t)}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

Each side of slab maintained at different temperature as following:

At t = 0, T = To

x = 0, T = To and x = L, T = Ts

and also find the eigen value of this temperature profile

Bonus (10 points):

a. What is benefit of "Sturm-Liouville System" in PDEs?
b. What is the difference between non-homogeneous and homogeneous boundary conditions?