PRINCE OF SONGKLA UNIVERSITY
FACULTY OF ENGINEERING

Midterm Examination: Semester II	Academic Year: 2003
Date: 26 December 2003	Time: 9:00-12:00
Subject: 240-552 Digital Signal Processing	Room: R300

Instructions:

This exam has 7 problems, 12 pages, and 84 points. Answer all questions on the exam sheets. You may use the back of the pages for scratch work. This exam is closed book and closed notes. No calculators will be allowed. You may consult one A4 sheet of notes (two sides).

Name:_____ Student code:_____

1 (21 pts) _____	4 (22 pts) _____
2 (9 pts) _____	5 (10 pts) _____
3 (8 pts) _____	6 (14 pts) _____

TOTAL _____

1. Consider the linear, time-invariant, discrete time system defined by the difference equation

$$y[n] - 4y[n-2] = x[n]$$

a) Find the general form of the homogeneous solution to this equation. (3 points)

b) Is the system causal? Why or why not? (3 points)

c) Is the system realizable? Why or why not? (3 points)

d) Let the input to the system be

$$x[n] = 0.1^n u[n]$$

Find the particular solution and then find the output $y[n]$
(Assume: $y[-1] = y[-2] = 0$). (12 points)

2. Sketch the convolution result of the following pairs of sequences (Please also label your sketch) (9 points)

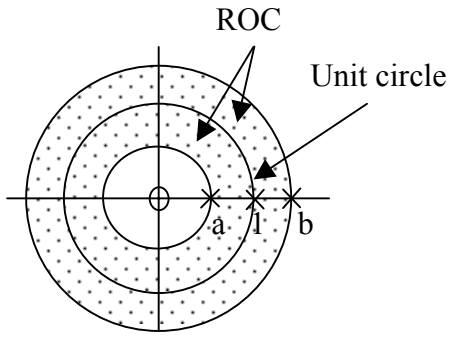
a) $u[n] * u[n+2]$

b) $(\delta[n]+\delta[n+1])+\delta[n-1]) * u[n]$

c) $(\delta[n]-\delta[n-1]) * u[n]$

3. Given $x[n] \overset{F}{\leftrightarrow} X\left(e^{j\omega}\right)$, prove that $x[-n] \overset{F}{\leftrightarrow} X\left(e^{-j\omega}\right)$, where x[n] is any real-valued sequence. (8 points)

4. Short answers:
   a) A pole-zero plot, and the region of convergence, of the z-transform of a sequence are shown. What conclusion(s) can you draw about the sequence? (3 points)



   b) A music signal has a bandwidth of 15,000 Hz and is to be transmitted digitally using a sampling rate of 20,000 Hz. Sketch the magnitude of the frequency response of the anti-aliasing filter that should be used in the continuous-to-discrete converter. Assume that the ideal filters are available. Label your sketch. (3 points)

   c) Suppose you need to upsample a discrete time signal by a factor of 60/27. An important design goal is that the sampling rate at any intermediate point should never be more than 10 times the sampling rate of the original signal. Also, no distortion should occur. Assuming ideal filters, sketch a block diagram of a purely discrete time system which can accomplish these goals, showing upsamplers and downsamplers (with sampling factors specified), and filters (assume ideal and specify cutoff frequency and gain). (5 points)

d) Find the frequency response $H(e^{j\omega})$ of the linear time invariant system whose input and output satisfy the difference equation

(4 points)

$$y[n] = 0.1y[n-1] + 0.7y[n-2] + 0.3y[n-4] - 0.6x[n] + 3x[n-2] + 2x[n-3]$$

e) Write a difference equation that characterizes a system whose frequency response is

(4 points)

$$H(e^{j\omega}) = \frac{1 - 0.4e^{-j\omega} + 0.1e^{-j4\omega}}{1 + 0.8e^{-j\omega} + 0.3e^{-j2\omega}}$$

5. A discrete time signal is defined by

$$h[n] = u[n] + 0.2^n u[n]$$

a) Find the z-transform, $H(z)$, of $h[n]$ (Also specify the ROC). (6 points)

b) Plot the poles and zeros of $H(z)$. (4 points)

6. The system function of a causal linear time-invariant system is

$$H(z) = \frac{1 - z^{-1}}{1 + \frac{3}{4} z^{-1}}$$

The input to the system is

$$x[n] = \left(\frac{1}{2}\right)^n u[n]$$

6.1 Find the impulse response of the system for all values of n. (4 points)
6.2 Find the output *y[n]* for all values of n. (8 points)
6.3 Is the system stable? (2 points)

Some common z-transform pairs

Sequence	Transform	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
$\delta[n-m]$	z^{-m}	All z except 0 (if $m>0$) or ∞ (if $m<0$)
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
$[\cos\omega_0 n]u[n]$	$\frac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
$[\sin\omega_0 n]u[n]$	$\frac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
$[r^n\cos\omega_0 n]u[n]$	$\frac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$
$[r^n\sin\omega_0 n]u[n]$	$\frac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$
$\begin{cases} a^n, & 0\le n\le N-1, \\ 0, & \text{otherwise} \end{cases}$	$\frac{1-a^N z^{-N}}{1-az^{-1}}$	$\lvert z\rvert > 0$