PRINCE OF SONGKLA UNIVERSITY
FACULTY OF ENGINEERING

Final Examination: Semester I — Academic Year: 2004

Date: October 2, 2004 — Time: 9:00-12:00

Subject: 230-601 Advanced Engineering Mathematics for Chemical Engineers — Room: หัวหุ่น

- อนุญาตให้นำเอกสารและเครื่องคำนวณทุกชนิดเข้าห้องสอบได้
- ทุจริตในการสอบโทษขั้นต่ำคือปรับตกในรายวิชาที่ทุจริตและพักการศึกษา 1 ภาคการศึกษา

Please do all 5 questions including bonus. Show all your work to receive full or partial credit. Total score is 120.

Question #	Total Score	Score
1	20	
2	20	
3	35	
4	30	
Bonus	15	
Total	**120**	

สุกฤทธิรา (บุญเรือง) รัตนวิไล

1. Steady-state heat balance on an elementary annular element of fin yields the equation **(20 points)**

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)-\frac{2h}{bk}(T-T_a)=0$$

Show the solution by using the modified Bessel equations.
Define: $y = T - T_a$

$$x = r\sqrt{\frac{2h}{bk}};$$ h, b, k are constants

2. Using Laplace Transform solve the differential equation. **(20 points)**

$$\frac{d^2y}{dt^2}+2t\frac{dy}{dt}-4y=1;\quad y(0)=0,\ y'(0)=0$$

3. A semi-infinite **<u>insulated rod</u>** has an initial and boundary conditions as follow:

$T(x,0) = To$
$T(\infty,t) = To$
$T(0,t) = f(t)$

Use Newton's law develop the partial differential equation describing the temperature of the semi-infinite rod as the function of time t and position x and **solve PDEs by using Laplace transform. (35 points)**

Define: $\theta(x,t) = T(x,t) - To$

4. Using **Separable Method of Variables** to find temperature profile of a slab T(y, t) based on the following conditions. **(30 points)**
A finite slab occupying the space between y = -b and y = +b is initially at temperature T_o. At time t = 0 the surfaces at y = ±b are suddenly raised to T_1 and maintained there.
The recommended dimensionless quantities are:

$$\text{dimensionless temperature} = \theta = \frac{T_1 - T}{T_1 - T_o}$$

$$\text{dimensionless length} = \eta = \frac{y}{b}$$

$$\text{dimensionless time} = \tau = \frac{\alpha^2 t}{b^2}$$

Bonus (15 points): Based on the chemical engineering problem that I assigned in class to find, one problem for each person, and solve it by using solution techniques for PDEs. **<u>Solve your own problem again but with different method for bonus points.</u>**