PRINCE OF SONGKLA UNIVERSITY
FACULTY OF ENGINEERING

Midterm Examination: Semester II — Academic Year: 2004
Date: 21December 2004 — Time: 13:30-16:30
Subject: 240-650 Principles of Pattern Recognition — Room: R300

Instructions:

This exam has 8 problems, 10 pages and 100 points. You may use the back of the pages for scratch work. This exam is open books and notes.

Name .... ID ....

1. Given $A = \begin{bmatrix} 3 & -3 \\ 3 & 2 \end{bmatrix}$, determine the eigenvalues of matrix A and eigenvectors associated with each eigenvalue. (10 points)

2. Suppose x is a random variable whose probability density function is defined by

$$p(x) = \begin{cases} 1/a & 0 < x < a \\ 0 & elsewhere \end{cases},$$

compute mean and variance of x. (10 points)

3. Box 1 contains 5 white, 10 yellow and 85 red balls. Box 2 contains 10 red, 20 yellow and 70 white balls. Box 2 contains 100 white balls. A ball is picked from a randomly selected box. If the ball is red, what is the probability that it came from Box 2? (15 points)

4. Suppose we have three categories in two dimensions with the following underlying distributions:

$$p(\mathbf{x} \mid \omega_1) \cong N(\mathbf{0}, \mathbf{I})$$

$$p(\mathbf{x} \mid \omega_2) \cong N\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \mathbf{I}\right)$$

$$p(\mathbf{x} \mid \omega_3) \cong \frac{1}{2} N\left(\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}, \mathbf{I}\right) + \frac{1}{2} N\left(\begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix}, \mathbf{I}\right)$$

with $P(\omega_i) = 1/3, i = 1,2,3$.

By explicit calculation of posterior probabilities, classify the point $x = \begin{bmatrix} 0.3 \\ 0.3 \end{bmatrix}$ for minimum probability of error. (15 points)

5. Suppose we have two normal distributions with the same covariance but different means: $N(\mu_1, \Sigma)$ and $N(\mu_2, \Sigma)$. In terms of their prior probabilities $P(\omega_1)$ and $P(\omega_2)$, state the condition that the Bayes decision boundary *not* pass between the two means. (10 points)

6. Consider two normal distributions in one dimension: $N(\mu_1,\sigma_1^2)$ and $N(\mu_2,\sigma_2^2)$. Imagine that we choose two random samples x_1 and x_2, one from each of the normal distributions and calculate their sum $x_3 = x_1+x_2$. Suppose we do this repeatedly.

a) Show that the distribution of x_3 is normal

b) What is the mean, μ_3?

c) What is the variance, σ_3^2 ? (15 points)

7. Let x have an exponential density

$$p(x \mid \theta) = \begin{cases} \theta e^{-\theta x} & x \geq 0 \\ 0 & otherwise \end{cases}.$$

Suppose that n samples $x1,\ldots,xn$, are drawn independently according to $p(x \mid \theta)$. Show that the maximum-likelihood estimate for θ is given by

$$\hat{\theta} = \frac{1}{\frac{1}{n}\sum_{k=1}^{n} x_k}.$$ (15 points)

8. Explain causes and effects of overfitting and underfitting. (10 points)