มหาวิทยาลัยสงขลานครินทร์

คณะวิศวกรรมศาสตร์

สอบปลายภาค ประจำภาคการศึกษา 2 ปีการศึกษา 2547

วันที่ 21 ธันวาคม 2547 เวลา 13.30 – 16.30.

220-486

วิชา Special Topic in Civil Engineering 3 (Introduction to Finite Element Method) ห้องสอบ A 401

ชื่อ-สกุล...

รหัส............................

คำชี้แจง

1. ข้อสอบทั้งหมดมี 5 ข้อ คะแนนรวม 100 คะแนน ดังแสดงในตารางข้างล่าง
2. ข้อสอบมีทั้งหมด 6 หน้า (รวมปก) ผู้สอบต้องตรวจสอบว่ามีครบทุกหน้าหรือไม่ (ก่อนลงมือทำ)
3. ให้ทำหมดทุกข้อลงในสมุดคำตอบ
4. ห้ามนำเอกสารใดๆ เข้าห้องสอบ **ทุจริตจะได้ E**
5. อนุญาตให้ใช้เครื่องคิดเลขได้ทุกชนิด
6. ห้ามหยิบ หรือยืมสิ่งของใดๆ ของผู้อื่นในห้องสอบ
7. อนุญาตให้นำ ***Class note*** เข้าห้องสอบได้
8. ***GOOD LUCK***

ตารางคะแนน

ข้อที่	คะแนนเต็ม	ได้
1	10	
2	15	
3	25	
4	25	
5	25	
รวม	100	

Problem 1 (10 Points)

The plate structure shown in Figure 1 is loaded and deformed in the plane of the figure. The applied load at D and the supports at N and I extend over a fairly narrow area. Given a list of what you think are the likely trouble spots that would require a locally finer finite element mesh to capture high stress gradients.

Identify those spots by its letter and reasons. For example, D: vicinity of point load

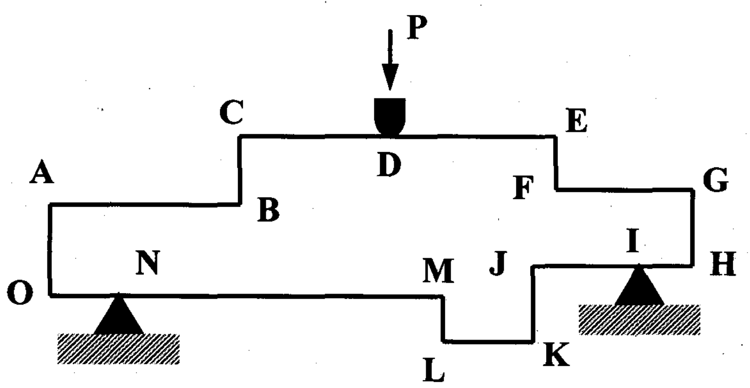

Figure 1

Problem 2 (15 Points)

Identify the symmetry and antisymmetry lines in the two-dimensional problems illustrated in Figure 2. They are (a) a circular disk under two diametrically opposite point forces (the famous Brazilian test for concrete); (b) the same disk under two diametrically opposite force pairs; (c) a clamped semi annulus under a force pair oriented as shown; (d) a stretched rectangular plate with a central circular hole. Finally (e) and (f) are half-planes under concentrated loads.

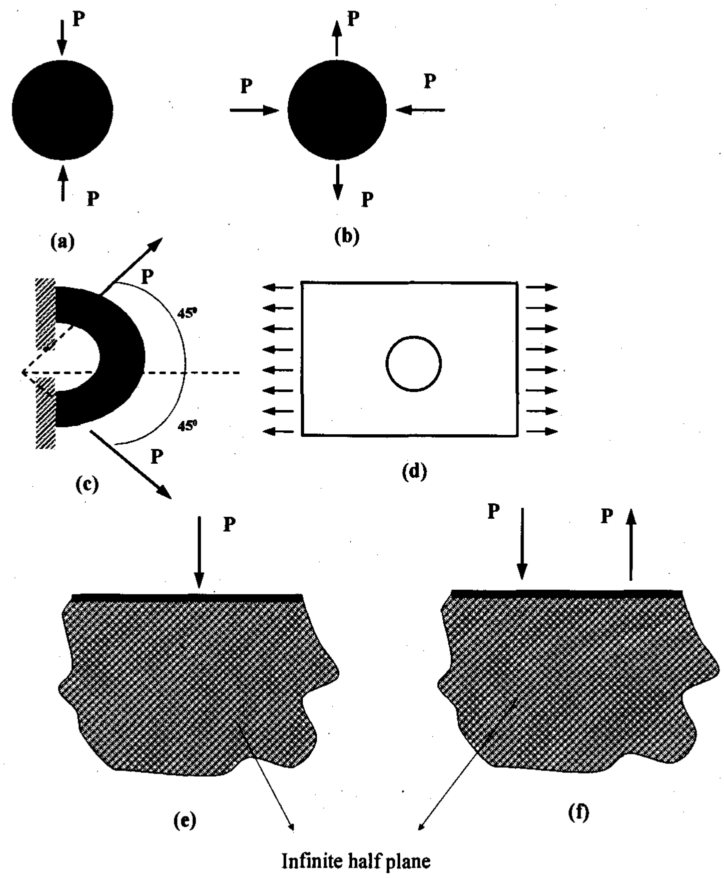

Figure 2

Problem 3 (25 Points)

Consider the two-element pin-jointed truss structure:

For each member, EA = 125

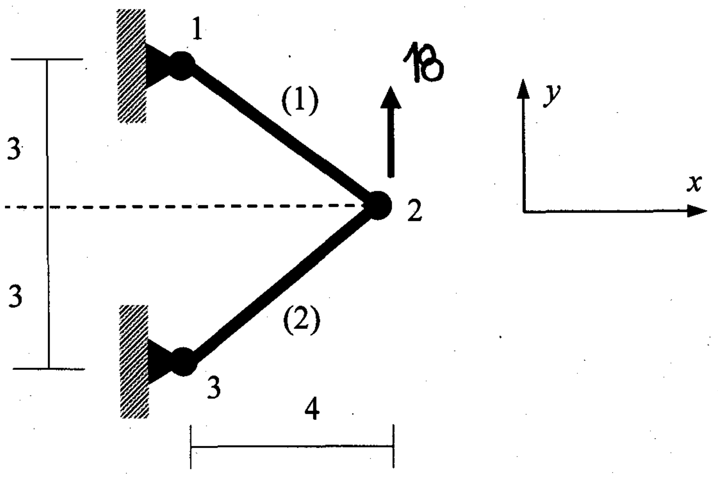

Figure 3

The element stiffness equations in global coordinates are

$$\begin{bmatrix} 16 & -12 & -16 & 12 \\ -12 & 9 & 12 & -9 \\ -16 & 12 & 16 & -12 \\ 12 & -9 & -12 & 9 \end{bmatrix} \begin{Bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{x2} \end{Bmatrix} = \begin{Bmatrix} f_{x1} \\ f_{y1} \\ f_{x2} \\ f_{x2} \end{Bmatrix}; element\ 1$$

$$\begin{bmatrix} 16 & 12 & -16 & -12 \\ 12 & 9 & -12 & -9 \\ -16 & -12 & 16 & 12 \\ -12 & -9 & 12 & 9 \end{bmatrix} \begin{Bmatrix} u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{x3} \end{Bmatrix} = \begin{Bmatrix} f_{x2} \\ f_{y2} \\ f_{x3} \\ f_{x3} \end{Bmatrix}; element\ 2$$

Items:

(a) Form the master stiffness equations and apply the loads and support conditions to get a reduced stiffness matrix and solve for the unknown displacements.

(b) Compute the axial force in element (1) from the computed nodal displacements.

(c) Show that an antisysmetry line can be identified to reduce the model of Figure 3 to just one member (pick any), and compute the displacement of node 2 using this reduced model.

Problem 4 (25 Points)

The hinge element depicted in Figure 4 is a Mechanics-of-Materials (MoM) member used in several FEM program. The degrees of freedom collected in $\overline{\mathbf{u}}$ are the two node rotations $\overline{\theta}_{zi}$ and $\overline{\theta}_{zj}$ about $\overline{z}$. The end moments, which compose $\overline{\mathbf{f}}$ are $\overline{m}_{zi}$ and $\overline{m}_{zj}$.

The member stiffness properties in the local system ($\overline{x}, \overline{y}, \overline{z}$) can be developed from the following relations. The deformation displacement equation is $\phi = \overline{\theta}_{zj} - \overline{\theta}_{zi}$, where ϕ is the hinge rotation. The constitutive equation is $M = k_H \phi$, where M is the internal moment taken up by the hinge and k_H is a known spring constant. The equilibrium equations at joint i and j are $\overline{m}_{zi} = -M$ and $\overline{m}_{zj} = M$

Construct the Tonti's Diagram and Element Stiffness for this hinge element.

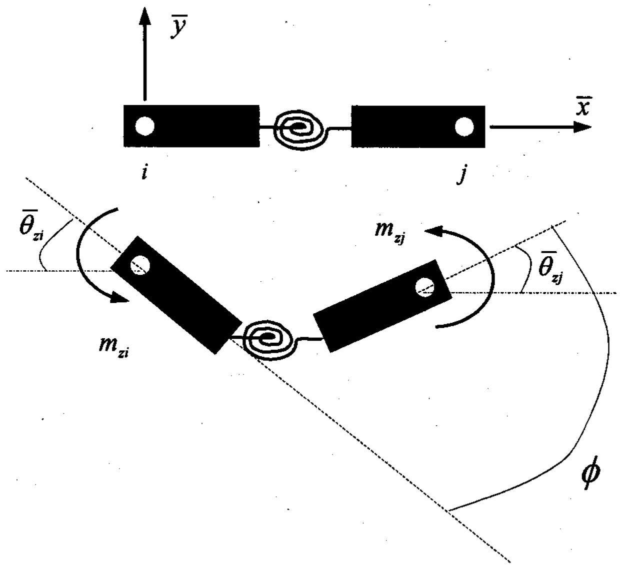

Figure 4

Problem 5 (25 Points)

For the quadratic bar element as shown in Figure 5, its element stiffness matrix is

$$\begin{pmatrix} \frac{7\,EA}{3\,L} & -\frac{8\,EA}{3\,L} & \frac{EA}{3\,L} \\ -\frac{8\,EA}{3\,L} & \frac{16\,EA}{3\,L} & -\frac{8\,EA}{3\,L} \\ \frac{EA}{3\,L} & -\frac{8\,EA}{3\,L} & \frac{7\,EA}{3\,L} \end{pmatrix}$$

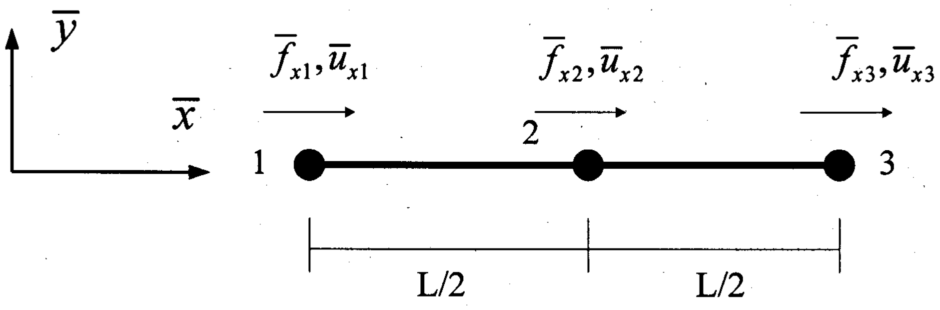

Figure 5

For $\dfrac{EA}{L} = 100$

(a) Determine its Eigenvalues and Eigenvectors.

(b) Give the physical meanings of these values.