ชื่อ.............................................................................................................รหัส....................................

# มหาวิทยาลัยสงขลานครินทร์

# คณะวิศวกรรมศาสตร์

การสอบปลายภาค ประจำภาคการศึกษาที่ 2 ปีการศึกษา 2547

วันที่ 25 กุมภาพันธ์ 2548 เวลา 13:30-15:30 น.

วิชา 215-612 ระเบียบวิธีไฟไนท์เอลิเมนท์ (Finite Element Method) ห้อง A201

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**<u>คำสั่ง</u>**

1. <u>ไม่อนุญาต</u>ให้นำหนังสือหรือเอกสารอื่นใดเข้าห้องสอบ
2. อนุญาตให้ใช้เครื่องคิดเลขได้ทุกรุ่น
3. ใช้ดินสอหรือปากกาทำข้อสอบก็ได้
4. ใช้เวลาทำ <u>2 ชั่วโมง</u>

| ทุจริตในการสอบ โทษต่ำสุด คือ พักการเรียน 1 ภาคการศึกษา และปรับตกในรายวิชาที่ทุจริต |
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**FINAL EXAM:**

ข้อสอบมีจำนวน 4 ข้อ ให้ทำทุกข้อ

ข้อ 1.________________ (20 คะแนน)

ข้อ 2.________________ (40 คะแนน)

ข้อ 3.________________ (20 คะแนน)

ข้อ 4.________________ (20 คะแนน)

รวม________________(100 คะแนน)

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1. (20 points)
During the finite element course we developed several elements. Fill out the table below for these elements.

| ELEMENT | List of Degrees of Freedom | Displacement function f(x) or f(x,y) | Sketch of Element (Label DOF) |
|---|---|---|---|
| SPAR in 2D space at angle to x-axis | | | |
| Beam along x-axis | | | |
| Frame at angle to x-axis | | | |
| 2D Solid-CST | | | |
| 2D Solid-LST | | | |

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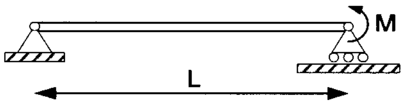
2. (40 points)

For a **PINNED-GUIDED** beam of length L, with a moment on the Right End, (at x=L), use the finite element formulation to solve for the **equation of deflection as a function of x in the beam.**



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3. (20 points)
Describe the concepts of **plane stress** and **plane strain** in detail.

a) Plane Stress (10 points)

b) Plane Strain (10 points)

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4. (20 points)
Use Gaussian quadrature with three Gauss points to evaluate the following integral:

$$T = \int_{-1}^{1} \frac{\cos(s)}{1+s^2} ds$$

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## ATTACHMENT

→ 2D SPAR (along x-axis) axial deflection

$$\text{Stiffness Matrix } [K] = \frac{AE}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} d_{1x} \\ d_{1y} \\ d_{2x} \\ d_{2y} \end{bmatrix}$$

→ 2D Bar (at angle θ to x-axis)
$C = \cos\theta$
$S = \sin\theta$

$$\text{Stiffness Matrix } [K] = \frac{AE}{L}\begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix} \qquad \begin{bmatrix} d_{1x} \\ d_{1y} \\ d_{2x} \\ d_{2y} \end{bmatrix}$$

→ Transformation of a vector from global d to local $\hat{d}$

(Local x-axis rotated counterclockwise by angle θ from global x-axis)

$$\begin{bmatrix} \hat{d}_x \\ \hat{d}_y \end{bmatrix} = \begin{bmatrix} C & S \\ -S & C \end{bmatrix}\begin{bmatrix} d_x \\ d_y \end{bmatrix}$$

→ 2D Beam (lateral deflection)

$$\text{Stiffness Matrix } [K] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \qquad \begin{bmatrix} d_{1y} \\ \phi_1 \\ d_{2y} \\ \phi_2 \end{bmatrix}$$

Shape Function of beam element of length L in bending

$$\begin{aligned} v(x) = {} & [(2/L^3)(d_{1y} - d_{2y}) + (1/L^2)(\phi_1 + \phi_2)]x^3 \\ & + [-(3/L^2)(d_{1y} - d_{2y}) - (1/L)(2\phi_1 + \phi_2)]x^2 \\ & + \phi_1 x + d_{1y} \end{aligned}$$

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→ Stiffness Matrix for Frame Element at angle θ to x-axis is

$$[K]=\frac{E}{L}\begin{bmatrix} AC^2+DS^2 & [A-D]CS & -6IS/L & -[AC^2+DS^2] & -[A-D]CS & -6IS/L \\ & AS^2+DC^2 & 6IC/L & -[A-D]CS & -[AS^2+DC^2] & 6IC/L \\ & & 4I & 6IS/L & -6IC/L & 2I \\ & & & AC^2+DS^2 & [A-D]CS & 6IS/L \\ & \text{Symmetry} & & & AS^2+DC^2 & -6IC/L \\ & & & & & 4I \end{bmatrix} \begin{bmatrix} d_{1x} \\ d_{1y} \\ \phi_1 \\ d_{2x} \\ d_{2y} \\ \phi_2 \end{bmatrix}$$

where $D=12I/L^2$ $C=\cos\theta$ $S=\sin\theta$

work-equivalent forces

$$\int w(x)v(x)dx = m_1\phi_1 + m_2\phi_2 + f_{1y}d_{1y} + f_{2y}d_{2y}$$

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Types of Elements:

| NAME | TYPE | DOF | REAL CONTANTS |
|---|---|---|---|
| LINK1 | 2D SPAR | UX, UY | A |
| BEAM3 | 2D BEAM | UX, UY, ROTZ | A, I, TK |
| BEAM4 | 3D BEAM | UX, UY, UZ, ROTX, ROTY, ROTZ | A, IZ, IY, IZ, TKY, TKZ |
| LINK8 | 3D SPAR | UX, UY, UZ | A |
| PIPE16 | 3D PIPE | UX, UY, UZ, ROTX, ROTY, ROTZ | OD, TK |
| PLANE42 | 2D SOLID (PLANE STRESS, PLANE STRAIN, AXISSYMMETRIC) | UX, UY | |
| SOLID45 | 3D SOLID | UX, UY, UZ | |
| SHELL63 | 3D PLATE/SHELL | UX, UY, UZ, ROTX, ROTY, ROTZ | TK |

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Pascal triangle

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & x & & y & & \\ & x^2 & & xy & & y^2 & \\ x^3 & & x^2y & & xy^2 & & y^3 \end{array}$$

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**Table D-1 Equivalent joint forces $f_0$ for different types of loads**

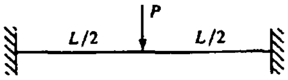
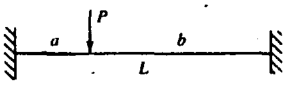
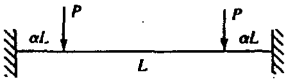
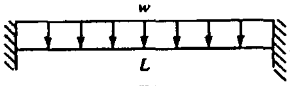
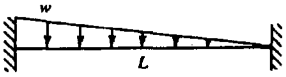
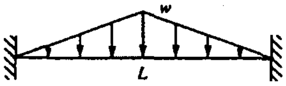
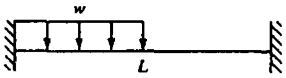
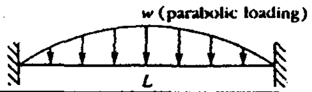
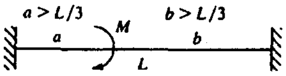
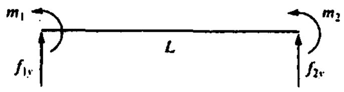
| | $f_{1y}$ | $m_1$ | Loading case | $f_{2y}$ | $m_2$ |
|---|---|---|---|---|---|
| 1. | $\frac{-P}{2}$ | $\frac{-PL}{8}$ |   | $\frac{-P}{2}$ | $\frac{PL}{8}$ |
| 2. | $\frac{-Pb^2(L+2a)}{L^3}$ | $\frac{-Pab^2}{L^2}$ |   | $\frac{-Pa^2(L+2b)}{L^3}$ | $\frac{Pa^2b}{L^2}$ |
| 3. | $-P$ | $-\alpha(1-\alpha)PL$ |   | $-P$ | $\alpha(1-\alpha)PL$ |
| 4. | $\frac{-wL}{2}$ | $\frac{-wL^2}{12}$ |   | $\frac{-wL}{2}$ | $\frac{wL^2}{12}$ |
| 5. | $\frac{-7wL}{20}$ | $\frac{-wL^2}{20}$ |   | $\frac{-3wL}{20}$ | $\frac{wL^2}{30}$ |
| 6. | $\frac{-wL}{4}$ | $\frac{-5wL^2}{96}$ |   | $\frac{-wL}{4}$ | $\frac{5wL^2}{96}$ |
| 7. | $\frac{-13wL}{32}$ | $\frac{-11wL^2}{192}$ |   | $\frac{-3wL}{32}$ | $\frac{5wL^2}{192}$ |

**Table D-1** *(Continued)*

| | $f_{1y}$ | $m_1$ | Loading case | $f_{2y}$ | $m_2$ |
|---|---|---|---|---|---|
| 8. | $\frac{-wL}{3}$ | $\frac{-wL^2}{15}$ |   | $\frac{-wL}{3}$ | $\frac{wL^2}{15}$ |
| 9. | $\frac{-M(a^2+b^2-4ab+L^2)}{L^3}$ | $\frac{Mb(2a-b)}{L^2}$ |   | $\frac{M(a^2+b^2-4ab+L^2)}{L^3}$ | $\frac{Ma(2b-a)}{L^2}$ |



Positive nodal force conventions

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Table for Gauss points for integration from minus one to one $\int_{-1}^{1} y(x)dx = \sum_{i=1}^{n} W_i y_i$ :

| Number of Points | Locations, $x_i$ | Associated Weights, $W_i$ |
|---|---|---|
| 1 | $x_1$=0.000 | 2.000 |
| 2 | $x_1$=+0.57735026918962<br>$x_2$=-0.57735026918962 | 1.000<br>1.000 |
| 3 | $x_1$=+0.77459666924148<br>$x_2$=0.000<br>$x_3$=-0.77459666924148 | 5/9=0.555…<br>8/9=0.888…<br>5/9=0.555… |
| 4 | $x_1$=+0.8611363116<br>$x_2$=+0.3399810436<br>$x_3$=-0.3399810436<br>$x_4$=-0.8611363116 | 0.3478548451<br>0.6521451549<br>0.6521451549<br>0.3478548451 |