**PRINCE OF SONGKLA UNIVERSITY**
**FACULTY OF ENGINEERING**

Midterm Examination: Semester I — Academic Year: 2005
Date: 3 August 2005 — Time: 9:00-12:00
Subject: 240-552 Digital Signal Processing — Room: A401

Instructions:

This exam has 7 problems, 13 pages, and 84 points. Answer all questions on the exam sheets. You may use the back of the pages for scratch work. This exam is closed book and closed notes. No calculators will be allowed. You may consult one A4 sheet of notes (two sides).

Name:________________________________________ Student code:____________

1 (15 pts) ______________ 2 (13 pts) ______________

3 (6 pts) ______________ 4 (10 pts) ______________

5 (19 pts) ______________ 6 (10 pts) ______________

7 (14 pts) ______________

TOTAL ____________

1. Given the following systems:

   System 1: $T(x[n]) = x[n] + x[n-1] + 2u[n+2]$

   System 2: $T(x[n]) = e^{x[n]}$

   System 3: $T(x[n]) = 0.5x[n] + 0.5x[-n]$

   a) Which system(s) is(are) linear? (3 points)

   b) Which system(s) is(are) causal? (3 points)

   c) Which system(s) is(are) BIBO stable? (3 points)

   d) Which system(s) is(are) memoryless? (3 points)

   e) Which system(s) is(are) time-invariant? (3 points)

2. Consider the linear, time-invariant, discrete time system defined by the difference equation

$$y[n]+0.3y[n-1]+0.02y[n-2]=x[n]$$

a) Find the general form of the homogeneous solution to this equation. (3 points)

b) Using DTFT to determine the impulse response of the system (10 points)

3. Sketch the convolution result of the following pairs of sequences (Please also label your sketch)

a) $(u[n]-u[n-5]) * (u[n+3] - u[n-3])$ (3 points)

b) $w[n]*v[n]$ where $w[n]=\begin{cases}0.5^n & 0\le n\le 4\\ 0 & elsewhere\end{cases}$ and $v[n]=\begin{cases}2 & |n|\le 10\\ 0 & elsewhere\end{cases}$ (3 points)

4. Given $x[n] \overset{F}{\leftrightarrow} X\left(e^{j\omega}\right)$, prove that $nx[n] \overset{F}{\leftrightarrow} j\frac{dX\left(e^{j\omega}\right)}{d\omega}$, where $\frac{d(\ )}{d\omega}$ is the derivative operator. (10 points)

5. Short answers:

a) Suppose you need to downsample a discrete time signal by a factor of 15/21. The goal is to design a downsampler with minimal lose of information. Assuming ideal filters are available, sketch a block diagram of a purely discrete time system which can accomplish these goals, showing upsamplers and downsamplers (with sampling factors specified), and filters (assume ideal and specify cutoff frequency). (5 points)

b) Find the frequency response $H(e^{j\omega})$ of the linear time invariant system whose input and output satisfy the difference equation

$$y[n] = 0.3y[n-1] + 0.2y[n-3] + 0.9y[n-4] - 0.5x[n] + 1.2x[n-1] + x[n-3]$$

(4 points)

c) Write a difference equation that characterizes a system whose frequency response is

(4 points)

$$H(e^{j\omega}) = \frac{1-0.3e^{-j2\omega}+0.1e^{-j3\omega}}{1+0.5e^{-j3\omega}+0.2e^{-j4\omega}}$$

d) Given $H(z) = \frac{1}{1+(0.2-0.3j)z^{-1}} + \frac{1}{1+(0.2+0.3j)z^{-1}} + \frac{1}{1-0.7z^{-1}}$, plot the poles and zeros of the system. What can you tell about the system from the given information? (for example, is the system stable?, is the system causal?)
(6 points)

6. A discrete time signal is defined by

$$h[n] = 0.2^n u[n] - 0.1^n u[-n-1]$$

a) Find the z-transform, $H(z)$, of $h[n]$ (Also specify the ROC). (6 points)

b) Plot the poles and zeros of $H(z)$. (4 points)

7 The system function of a causal linear time-invariant system is

$$H(z) = \frac{z^{-2}}{1 + 0.1z^{-1}}$$

The input to the system is $x[n] = (0.5)^n u[n]$

6.1 Find the impulse response of the system for all values of $n$. (4 points)
6.2 Find the output $y[n]$ for all values of $n$. (8 points)
6.3 Is the system stable? (2 points)

**Some common z-transform pairs**

| Sequence | Transform | ROC |
|---|---|---|
| $\delta[n]$ | $1$ | All $z$ |
| $u[n]$ | $\dfrac{1}{1-z^{-1}}$ | $\lvert z\rvert>1$ |
| $-u[-n-1]$ | $\dfrac{1}{1-z^{-1}}$ | $\lvert z\rvert<1$ |
| $\delta[n-m]$ | $z^{-m}$ | All $z$ except 0 (if $m>0$) or $\infty$ (if $m<0$) |
| $a^n u[n]$ | $\dfrac{1}{1-az^{-1}}$ | $\lvert z\rvert>\lvert a\rvert$ |
| $-a^n u[n]$ | $\dfrac{1}{1-az^{-1}}$ | $\lvert z\rvert<\lvert a\rvert$ |
| $na^n u[n]$ | $\dfrac{az^{-1}}{(1-az^{-1})^2}$ | $\lvert z\rvert>\lvert a\rvert$ |
| $-na^n u[-n-1]$ | $\dfrac{az^{-1}}{(1-az^{-1})^2}$ | $\lvert z\rvert<\lvert a\rvert$ |
| $[\cos\omega_0 n]u[n]$ | $\dfrac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$ | $\lvert z\rvert>1$ |
| $[\sin\omega_0 n]u[n]$ | $\dfrac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$ | $\lvert z\rvert>1$ |
| $[r^n\cos\omega_0 n]u[n]$ | $\dfrac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$ | $\lvert z\rvert>r$ |
| $[r^n\sin\omega_0 n]u[n]$ | $\dfrac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$ | $\lvert z\rvert>r$ |
| $\begin{cases} a^n, & 0\le n\le N-1, \\ 0, & \text{otherwise} \end{cases}$ | $\dfrac{1-a^N z^{-N}}{1-az^{-1}}$ | $\lvert z\rvert>0$ |