PRINCE OF SONGKLA UNIVERSITY
FACULTY OF ENGINEERING

Final Examination: Semester I — Academic Year: 2005

Date: October 5, 2005 — Time: 9:00-12:00

Subject: 230-601 Advanced Engineering Mathematics for Chemical Engineers — Room: R300

- อนุญาตให้นำเอกสารและเครื่องคำนวณทุกชนิดเข้าห้องสอบได้
- ทุจริตในการสอบโทษขั้นต่ำคือปรับตกในรายวิชาที่ทุจริตและพักการศึกษา 1 ภาคการศึกษา

Please do all 4 questions including bonus. Show all your work to receive full or partial credit. Total score is 100.

This exam has totally 4 pages.

Question #	Total Score	Score
1	20	
2	30	
3	20	
4	30	
Total	100	

สุกฤทธิรา รัตนวิไล

1. An expression for the steady state temperature in a solid cylinder is following.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}=0; \quad 0 \le r \le b \text{ and } 0 \le z \le c$$

When the boundary at z = 0 is kept at temperature F(r). Furthermore, there is convection into medium at zero temperature from the surfaces r = b and z = C obtains expressions as follows:

$$\left.\frac{\partial T}{\partial r}\right|_{r=b}+hr\big|_{r=b}=0; \qquad \left.\frac{\partial T}{\partial z}\right|_{z=c}+hz\big|_{z=c}=0$$

Find temperature distribution T(r, z) in a solid cylinder. (20 points)

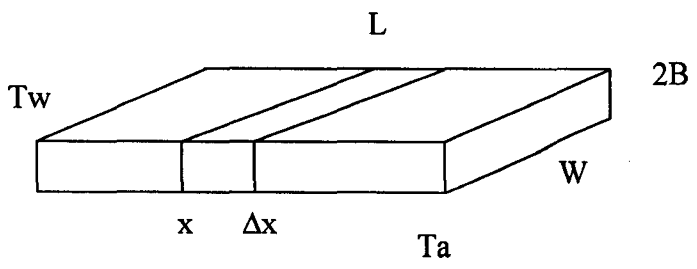
2. 2.1 Apply the method of Frobenius method for solving the differential equation. (10 points)

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-x^2y=0$$

2.2 Using Laplace Transform solve the differential equation. (10 points)

$$\frac{d^2y}{dt^2}-2\frac{dy}{dt}+y=e^t+t; \qquad y(0)=1, \qquad \left.\frac{dy}{dt}\right|_{t=0}=0$$

2.3 A straight rectangular finite fin with uniform cross section is used to cooling a hot surface of wall. One side of fin is maintained temperature known to be Tw. The ambient air flowing around the fin has a temperature of Ta. The fin conductivity takes a value of k. The heat transfer coefficient around the surface and fin density are constant h and ρ, respectively. Heat capacity of fin is equal C_p. **Show the differential equation** for the case when fin temperature changes mainly in the x direction and different time (t). (10 points)



3. A semi-infinite string has an initial and boundary conditions as follow:

$$W(x,0) = 0; \qquad W(\infty,t) = 0$$

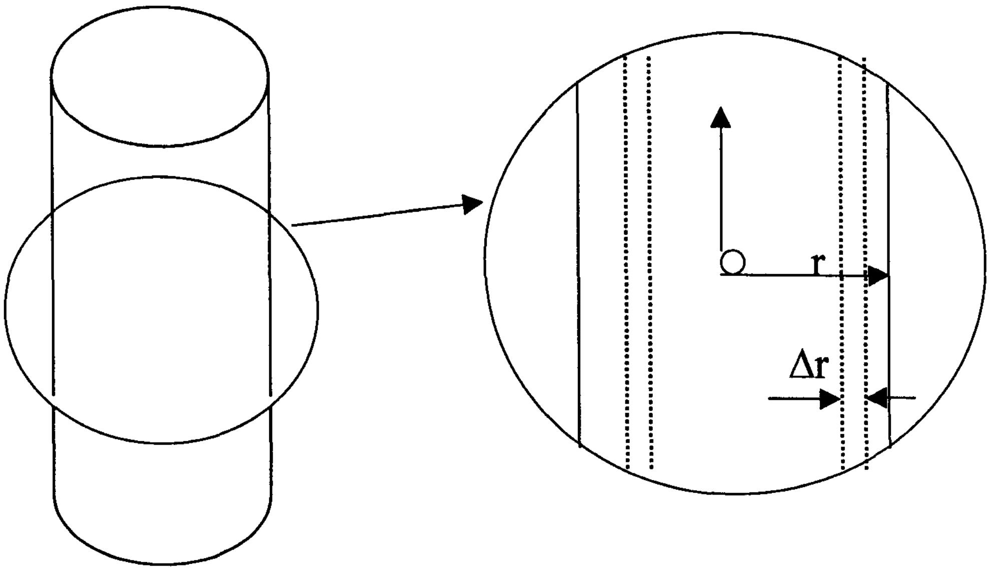
$$W(0,t) = f(t); \qquad \left.\frac{\partial W}{\partial t}\right|_{t=0} = 0$$

Appropriate displacement equation is given by:

$$\frac{\partial^2 W}{\partial t^2} = c^2 \frac{\partial^2 W}{\partial x^2}$$

Find the displacement W(x,t) of an elastic string subject to the above conditions **by using Laplace transform.** (20 points)

4. The diffusion of a solute into an infinite cylinder as shown in Figure 4. The cylinder initially contains no solute. At time zero, it is suddenly immersed in a well-stirred solution that is of such enormous volume that its solute concentration is constant. The solute diffuses into the cylinder symmetrically. Problem like this are important in the chemical treatment of wood.



4.1 Show that the appropriate mass balance is given by (5 points)

$$\frac{\partial C}{\partial t} = \frac{D}{r}\frac{\partial}{\partial r} r \frac{\partial C}{\partial r} \quad ....(1)$$

D = diffusion coefficient
C = solute's concentration
t = time

4.2 Define three new variables,

$$\text{dimensionless concentration}: \theta = 1 - \frac{C}{C(\text{surface})}$$

$$\text{dimensionless position}: \xi = \frac{r}{R_o}$$

$$\text{dimensionless time}: \tau = \frac{Dt}{R_0^{\,2}}$$

Show that equation (1) becomes (5 points)

$$\frac{\partial \theta}{\partial \tau} = \frac{1}{\xi}\frac{\partial}{\partial \xi}\xi\frac{\partial \theta}{\partial \xi} \quad \text{................(2)}$$

4.3 Find the solute's concentration in this cylinder as a function of time and location in r direction by using **separation of variables** and boundary conditions as follow: (20 points)

$t \leq 0$,	all r,	$C = 0$
$t > 0$,	$r = Ro$,	$C = C(\text{surface})$
$t > 0$,	$r = 0$,	$\frac{\partial C}{\partial r} = 0$