

1. Suppose a cellular telephone is equally likely to make zero handoffs (H_0), one handoff (H_1), or more than one handoff (H_2). Also, a caller is either on foot (F) with probability $5/12$ or in a vehicle (V). Suppose we also learn that $1/4$ of all callers are on foot making calls with no handoffs and that $1/6$ of all callers are vehicle users making calls with a single handoff. Given these additional facts, *find all possible ways to fill in the table of probabilities*(e.g. find $p_0, p_1, p_2, q_0, q_1, q_2$). (6 marks)

Probability table

| | H_0 | H_1 | H_2 |
|---|-------|-------|-------|
| F | p_0 | p_1 | p_2 |
| V | q_0 | q_1 | q_2 |

Answer _____

2. . Given a packet is corrupted with probability ϵ . From the following applications, *give the name of random variable* (3marks)

(a) Let Y denote the number of packet received in error out of 100 packets transmitted. Y has the PMF

$$P_Y(y) = \begin{cases} \binom{100}{y} (\epsilon)^y (1 - \epsilon)^{100-y} & y = 0,1,\dots,100 \\ 0 & \text{otherwise} \end{cases}$$

The expected values of Y is $E[Y] = 100\epsilon$

Answer _____

(b) If packet arrivals with an average arrival rate of 1000 packets per second, then the number N of packets that arrive in 5 seconds has the PMF

$$P_N(n) = \begin{cases} 5000^n e^{-5000} / n! & n = 0, 1, \dots \\ 0 & \text{otherwise} \end{cases}$$

The expected value of N is $E[N] = 5000$

Answer _____

(c) Let L equal the number of packets that must be received to decode 5 packets in error. L has the PMF

$$P_L(l) = \begin{cases} \binom{l-1}{4} \epsilon^5 (1-\epsilon)^{l-5} & l = 5, 6, \dots \\ 0 & \text{otherwise} \end{cases}$$

The expected value of L is $E[L] = 5/\epsilon$

Answer _____

3. In a packet voice communications system, a source transmits packets containing digitized speech to a receiver. Because transmission errors occasionally occur, an acknowledgment (ACK) or a nonacknowledgment (NAK) is transmitted back to the source to indicate the status of each received packet. When the transmitter gets a NAK, the packet is retransmitted. Voice packets are delay sensitive and a packet can be transmitted a maximum of d times. If a packet transmission is an independent Bernoulli trial with success probability p , answer the following questions.

What is the PMF of T , the number of times a packet is transmitted? (3 marks)

Answer _____

4. A source wishes to transmit data packets to a receiver over a radio link. The receiver uses error detection to identify packets that have been corrupted by radio noise. When a packet is received error-free, the receiver sends an acknowledgment (ACK) back to the source. When the receiver gets a packet with errors, a negative acknowledgment (NAK) message is sent back to the source. Each time the source receives a NAK, the packet is retransmitted. We assume that each packet transmission is independently corrupted by errors with probability q .

- (a) Find the PMF of X , the number of times that packet is transmitted by the source
(3 marks)

Answer _____

- (b) Suppose each packet takes 1 millisecond to transmit and that the source waits an additional millisecond to receive the acknowledgment message (ACK or NAK) before retransmitting. Let T equal the time required until the packet is successfully received. What is the relationship between T and X ? (1 marks)

Answer _____

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(c) From question 4(a) *what is the PMF of T if $T = 2X - 1$?* (3 marks)

Answer _____

(d) *Find the expected values of the random variable T* (5 marks)

Given $\sum_{x=1}^{\infty} q^x = \frac{1}{1-q}$ and $\sum_{x=1}^{\infty} xq^x = \frac{q}{(1-q)^2}$ when $|q| < 1$

Answer _____

6. Random variable X has the PDF:

$$f_X(x) = \begin{cases} c \frac{x}{2} e^{-x/2} & x \geq 0 \\ 0 & \textit{otherwise} \end{cases}$$

Find the following: (10 marks)

(a) The constant c . (4 marks)

Answer _____

(b) The CDF $F_X(x)$. (4 marks)

Answer _____

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(c) $P[0 \leq X \leq 6]$ (1 mark)

Answer _____

(d) $P[-4 \leq X \leq 4]$ (1 mark)

Answer _____

7. The peak temperature T , as measured in degrees Fahrenheit, on an April day in Bangkok is the Gaussian $(85, 10)$ random variable. Find the following: (10 marks)

| z | $\Phi(z)$ |
|-----|-----------|
| 1 | 0.841 |
| 1.5 | 0.933 |
| 2 | 0.977 |
| 2.5 | 0.993 |

(a) $P[T > 100]$ (2 marks)

Answer _____

(b) $P[T < 60]$ (2 marks)

Answer _____

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(c) $P[70 \leq T \leq 100]$ (3 marks)

Answer _____

(d) Use information from questions above, sketch *the PDF* $f_T(t)$ showing *the center of the bell*, the area under the bell and the height of the peak. (3 marks)

Answer _____

