PRINCE OF SONGKLA UNIVERSITY
FACULTY OF ENGINEERING

Midterm Examination: Semester II | Academic Year: 2005
Date: 17December 2005 | Time: 9:00-12:00
Subject: 240-650 Principles of Pattern Recognition | Room: R200

Instructions:

This exam has 6 problems, 12 pages and 85 points. You may use the back of the pages for scratch work. This exam is open books and notes.

Name .... ID ....

1. Suppose $\Sigma = \begin{bmatrix} 2 & 1 \\ -5 & 4 \end{bmatrix}$ is a covariance matrix that represents the distribution of a set of data. If we want to perform the *whitening transformation* on this set of data, what is the transformation matrix? (20 points)
Hint: We need to normalize the eigenvectors.

2. Suppose x is a random variable whose probability density function is defined by

$$p(x) = \begin{cases} e^{-a/4} & 0 < x < a \\ 0 & elsewhere \end{cases},$$

compute the mean and variance of x. (10 points)

3. Box 1 contains 10 red balls and 20 blue balls. Box 2 contains 30 red balls and 5 blue balls. Box 3 contains 100 yellow balls. If a ball is randomly selected from the boxes, what is the probability that the selected ball is red? If a ball is randomly selected from Box 1 and put it in Box 3, then a ball is randomly selected from one of the 3 boxes. What is the probability that the selected ball is blue and comes from Box 3.
(15 points)

4. Suppose we have three categories in two dimensions with the following underlying distributions:

$$p(\mathbf{x} \mid \omega_1) \cong N(\mathbf{0}, \mathbf{I})$$

$$p(\mathbf{x} \mid \omega_2) \cong N\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \mathbf{I}\right)$$

$$p(\mathbf{x} \mid \omega_3) \cong \frac{3}{4} N\left(\begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix}, \mathbf{I}\right) + \frac{1}{4} N\left(\begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix}, \mathbf{I}\right)$$

with $P(\omega_i) = 1/3, i = 1,2,3.$

By explicit calculation of posterior probabilities, classify the point $x = \begin{bmatrix} 0.3 \\ 0.5 \end{bmatrix}$ for minimum probability of error. (15 points)

5. Suppose a random variable x is normally distributed and $y = 3x + 10$. Prove that y is also normally distributed. (10 points)

6. Let x have an exponential density

$$p(x|\theta) = \begin{cases} \theta e^{-\theta x} & x \geq 0 \\ 0 & otherwise \end{cases}.$$

Suppose that n samples $x_1, \ldots, x_n$, are drawn independently according to $p(x|\theta)$. Show that the maximum-likelihood estimate for θ is given by

$$\hat{\theta} = \frac{1}{\frac{1}{n}\sum_{k=1}^{n} x_k}.$$ (15 points)