Name.....Student I.D.....

Department of Mining and Materials Engineering

Faculty of Engineering

Prince of Songkla University

Mid-term Examination for Semester: 2 Academic Year: 2005

Date: December 13, 2005 Time: 9.00-12.00

Subject: 237-508 Structures and Mechanical Properties of Materials Room: R300

Instruction

1. There are 3 problem sets. Please do all of them and write your answers in the space provided. If you need more space, you can write on the back of paper.
2. Only two (2) pieces of A4-size note are allowed. You may write on both sides of them. Please return them with your answers.
3. Dictionary, calculator, and stationery are also allowed.
4. Text books and other studying materials are not allowed.
5. This mid-term exam is counted for 30% of the total grade.

Dr. Thawatchai Plookphol

Problem No.	Full Score (points)	Student's Score (points)
1.	60	
2.	40	
3.	50	
Total	150	

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1. The 3-D state of stress is given by:

$$\sigma_{ij} = \begin{bmatrix} 25 & 0 & 0 \\ 0 & 7 & -3\sqrt{3} \\ 0 & -3\sqrt{3} & 13 \end{bmatrix} \text{ MPa}$$

(a) Calculate the three invariants of stress (I_1, I_2, I_3) (15 points)

(b) Calculate the principal stresses ($\sigma_1, \sigma_2, \sigma_3$). Please show your work. (30 points)

(c) Write a new stress tensor σ'_{ij} from the principal stresses (part (b)) in the form given below, using the convention that $\sigma_1 > \sigma_2 > \sigma_3$. **Calculate the invariants of the stress tensor** σ'_{ij} **and compare with the answer in part (a).** (15 points)

$$\sigma'_{ij} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} \text{ MPa}$$

Given:

$$det\begin{bmatrix} \sigma_{11}-\sigma & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22}-\sigma & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33}-\sigma \end{bmatrix} = 0$$

$$I_3 - \sigma I_2 + \sigma^2 I_1 - \sigma^3 = 0$$

where,

$$I_1 = \sigma_{11} + \sigma_{22} + \sigma_{33}$$

$$I_2 = (\sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21}) + (\sigma_{22}\sigma_{33} - \sigma_{23}\sigma_{32}) + (\sigma_{11}\sigma_{33} - \sigma_{13}\sigma_{31})$$

$$I_3 = \sigma_{11}(\sigma_{22}\sigma_{33} - \sigma_{23}\sigma_{32}) + \sigma_{21}(\sigma_{13}\sigma_{32} - \sigma_{12}\sigma_{33}) + \sigma_{31}(\sigma_{12}\sigma_{23} - \sigma_{13}\sigma_{22})$$

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2. A three-dimensional state of stress is given by:

$$\sigma_{11} = +80 \text{ MPa} \qquad \sigma_{12} = +20 \text{ MPa}$$
$$\sigma_{22} = -40 \text{ MPa} \qquad \sigma_{23} = +30 \text{ MPa}$$
$$\sigma_{33} = +60 \text{ MPa} \qquad \sigma_{31} = -50 \text{ MPa}$$

Show that the total stress on a plane with direction cosines

$$n_1 = \frac{1}{\sqrt{2}} \qquad n_2 = \frac{1}{2} \qquad n_3 = -\frac{1}{2}$$

is 106.5 MPa and that the normal and shear stresses on this plane are 79.5 MPa and 71.0 MPa respectively. (40 points)

Given:

$$\overline{S} = \overline{\overline{\sigma}} \cdot \hat{n} \qquad \text{or} \qquad S_i = \sum_{j=1}^{3} \sigma_{ij} n_j$$

$$S^2 = S_1^2 + S_2^2 + S_3^2$$

$$\sigma = S_1 \cdot n_1 + S_2 \cdot n_2 + S_3 \cdot n_3$$

$$S^2 = \sigma^2 + \tau^2$$

where,

S = Total stress acting on the plane,
σ = Normal stress acting on the plane,
τ = Shear stress acting on the plane,
n_1, n_2, n_3 are direction cosines.

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3. A copper single crystal (cubic) experiences the state of stress of

$$\sigma_{ij} = \begin{bmatrix} 10 & -8 & 4 \\ -8 & 20 & 0 \\ 4 & 0 & 5 \end{bmatrix} \text{ MPa}$$

Some elastic constants of copper are given by:

$$E_{11} = 66.7 \text{ GPa}, \qquad G_{12} = 75.2 \text{ GPa}, \qquad \nu_{12} = 0.42$$

(a) Determine the compliance matrix $[S]$ for the copper single crystal (i.e. compliance tensor) (20 points)
(b) Determine the engineering strain in the copper sample by assuming deformation is linear elastic. (30 points)

Given: $S_{11} = \frac{1}{E_{11}}, \qquad S_{12} = -\frac{\nu_{12}}{E_{11}}, \qquad S_{44} = \frac{1}{G_{12}}$

$$[\varepsilon] = [S][\sigma] \qquad \text{or}$$

$$\varepsilon_{ij} = S_{ijkl}\sigma_{kl}$$

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