ชื่อ...รหัส....................................

มหาวิทยาลัยสงขลานครินทร์

คณะวิศวกรรมศาสตร์

การสอบปลายภาค ประจำภาคการศึกษาที่ 2 ปีการศึกษา 2548

วันที่ 5 มีนาคม 2549 เวลา 13.30-16.30 น.

วิชา 215-612 ระเบียบวิธีไฟไนท์เอลิเมนท์ (Finite Element Method) ห้อง หัวหุ่น

คำสั่ง

1. ไม่อนุญาตให้นำหนังสือหรือเอกสารอื่นใดเข้าห้องสอบ
2. อนุญาตให้ใช้เครื่องคิดเลขได้ทุกรุ่น
3. ใช้ดินสอหรือปากกาทำข้อสอบก็ได้
4. ใช้เวลาทำ 3 ชั่วโมง

ทุจริตในการสอบ โทษต่ำสุด คือ พักการเรียน 1 ภาคการศึกษา และปรับตกในรายวิชาที่ทุจริต

FINAL EXAM:

ข้อสอบมีจำนวน 5 ข้อ ให้ทำทุกข้อ

ข้อ 1.______________ (20 คะแนน)

ข้อ 2.______________ (30 คะแนน)

ข้อ 3.______________ (30 คะแนน)

ข้อ 4.______________ (20 คะแนน)

ข้อ 5.______________ (20 คะแนน)

รวม______________(120 คะแนน)

พฤทธิ์ธร สมิตไมตรี

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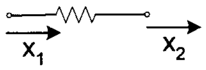
1. (20 points)
During the finite element course we developed several elements. Fill out the table below for these elements.

ELEMENT	List of Degrees of Freedom	Displacement function f(x) or f(x,y)	Sketch of Element (Label DOF)
"For example" Spring in 1D	x_1, x_2	$f(x)=a_1+a_2x$	
BAR in 2D space at angle to x-axis			
Beam along x-axis			
Frame at angle to x-axis			
2D Solid-CST			
2D Solid-LST			

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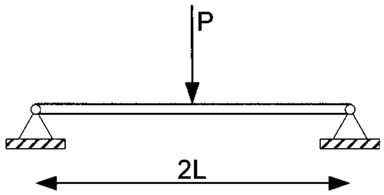
2. (30 points)
For a **PINNED-PINNED** beam of length 2L,modulus E, moment of inertia I, with a point load P at the center, use the finite element formulation to develop the matrix representation and solve for the **deflection at the center of the beam**



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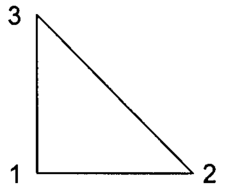
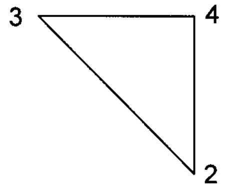
3. (30 points) The stiffness matrices for two CST elements are given in terms of constants a, b, c, … and A, B, C, … etc. These elements are to be assembled into a square block 1-2-4-3 (x-axis is horizontal)

$$[K]^{(1)} = \begin{bmatrix} a & b & -c & -d & -e & -g \\ b & f & -g & -h & -d & -i \\ -c & -g & c & 0 & 0 & g \\ -d & -h & 0 & h & d & 0 \\ -e & -d & 0 & g & e & 0 \\ -g & -i & g & 0 & 0 & i \end{bmatrix} \quad \begin{Bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ x_3 \\ y_3 \end{Bmatrix}$$



$$[K]^{(2)} = \begin{bmatrix} A & B & -C & -D & -E & -G \\ B & F & -G & -H & -D & -I \\ -C & -G & C & 0 & 0 & G \\ -D & -H & 0 & H & D & 0 \\ -E & -D & 0 & G & E & 0 \\ -G & -I & G & 0 & 0 & I \end{bmatrix} \quad \begin{Bmatrix} x_2 \\ y_2 \\ x_3 \\ y_3 \\ x_4 \\ y_4 \end{Bmatrix}$$



What are the force components on node 3 resulting from a unit displacement, in the x direction, of node 2 of the assembled structure, with all other degrees of freedom fixed to zero?

4. (20 points)
Describe the concepts of **plane stress** and **plane strain** in detail.

a) Plane Stress (10 points)

b) Plane Strain (10 points)

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5. (20 points)
Use Gaussian quadrature with three Gauss points to evaluate the following integral:

$$T = \int_{-1}^{1} \frac{\sin(s)}{s^2 + 2} ds$$

Bonus Question: Is there any wrong?, why do you get the answer like that?

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ATTACHMENT

→ 2D SPAR (along x-axis) axial deflection

$$\text{Stiffness Matrix } [K] = \frac{AE}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} d_{1x} \\ d_{1y} \\ d_{2x} \\ d_{2y} \end{bmatrix}$$

→ 2D Bar (at angle θ to x-axis)

$C = \cos\theta$

$S = \sin\theta$

$$\text{Stiffness Matrix } [K] = \frac{AE}{L}\begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix} \qquad \begin{bmatrix} d_{1x} \\ d_{1y} \\ d_{2x} \\ d_{2y} \end{bmatrix}$$

→ Transformation of a vector from global d to local $\hat{d}$

(Local x-axis rotated counterclockwise by angle θ from global x-axis)

$$\begin{bmatrix} \hat{d}_x \\ \hat{d}_y \end{bmatrix} = \begin{bmatrix} C & S \\ -S & C \end{bmatrix}\begin{bmatrix} d_x \\ d_y \end{bmatrix}$$

→ 2D Beam (lateral deflection)

$$\text{Stiffness Matrix } [K] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \qquad \begin{bmatrix} d_{1y} \\ \phi_1 \\ d_{2y} \\ \phi_2 \end{bmatrix}$$

Shape Function of beam element of length L in bending

$$\begin{aligned} v(x) = {} & [(2/L^3)(d_{1y} - d_{2y}) + (1/L^2)(\phi_1 + \phi_2)]x^3 \\ & + [-(3/L^2)(d_{1y} - d_{2y}) - (1/L)(2\phi_1 + \phi_2)]x^2 \\ & + \phi_1 x + d_{1y} \end{aligned}$$

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→ Stiffness Matrix for Frame Element at angle θ to x-axis is

$$[K]=\frac{E}{L}\begin{bmatrix} AC^2+DS^2 & [A-D]CS & -6IS/L & -[AC^2+DS^2] & -[A-D]CS & -6IS/L \\ & AS^2+DC^2 & 6IC/L & -[A-D]CS & -[AS^2+DC^2] & 6IC/L \\ & & 4I & 6IS/L & -6IC/L & 2I \\ & & & AC^2+DS^2 & [A-D]CS & 6IS/L \\ & \text{Symmetry} & & & AS^2+DC^2 & -6IC/L \\ & & & & & 4I \end{bmatrix} \begin{bmatrix} d_{1x} \\ d_{1y} \\ \phi_1 \\ d_{2x} \\ d_{2y} \\ \phi_2 \end{bmatrix}$$

where $D=12I/L^2$ $C=\cos\theta$ $S=\sin\theta$

work-equivalent forces

$$\int w(x)v(x)dx = m_1\phi_1 + m_2\phi_2 + f_{1y}d_{1y} + f_{2y}d_{2y}$$

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Types of Elements:

NAME	TYPE	DOF	REAL CONTANTS
LINK1	2D SPAR	UX, UY	A
BEAM3	2D BEAM	UX, UY, ROTZ	A, I, TK
BEAM4	3D BEAM	UX, UY, UZ, ROTX, ROTY, ROTZ	A, IZ, IY, IZ, TKY, TKZ
LINK8	3D SPAR	UX, UY, UZ	A
PIPE16	3D PIPE	UX, UY, UZ, ROTX, ROTY, ROTZ	OD, TK
PLANE42	2D SOLID	UX, UY (PLANE STRESS, PLANE STRAIN, AXISSYMMETRIC)	
SOLID45	3D SOLID	UX, UY, UZ	
SHELL63	3D PLATE/SHELL	UX, UY, UZ, ROTX, ROTY, ROTZ	TK

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Pascal triangle

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & x & & y & & \\ & x^2 & & xy & & y^2 & \\ x^3 & & x^2y & & xy^2 & & y^3 \end{array}$$

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Table D–1 Equivalent joint forces f_0 for different types of loads

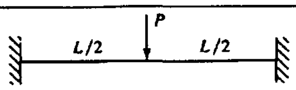
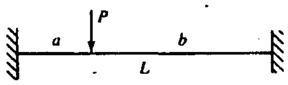
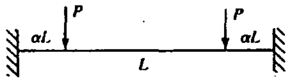
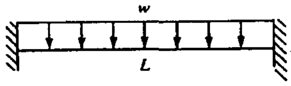
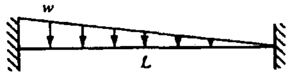
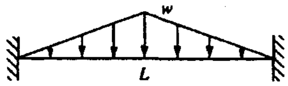
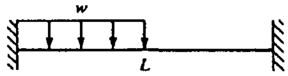
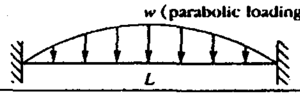
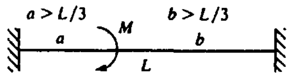
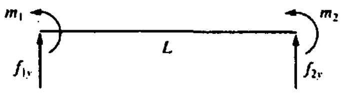
	f_{1y}	m_1	Loading case	f_{2y}	m_2
1.	$\frac{-P}{2}$	$\frac{-PL}{8}$		$\frac{-P}{2}$	$\frac{PL}{8}$
2.	$\frac{-Pb^2(L+2a)}{L^3}$	$\frac{-Pab^2}{L^2}$		$\frac{-Pa^2(L+2b)}{L^3}$	$\frac{Pa^2b}{L^2}$
3.	$-P$	$-\alpha(1-\alpha)PL$		$-P$	$\alpha(1-\alpha)PL$
4.	$\frac{-wL}{2}$	$\frac{-wL^2}{12}$		$\frac{-wL}{2}$	$\frac{wL^2}{12}$
5.	$\frac{-7wL}{20}$	$\frac{-wL^2}{20}$		$\frac{-3wL}{20}$	$\frac{wL^2}{30}$
6.	$\frac{-wL}{4}$	$\frac{-5wL^2}{96}$		$\frac{-wL}{4}$	$\frac{5wL^2}{96}$
7.	$\frac{-13wL}{32}$	$\frac{-11wL^2}{192}$		$\frac{-3wL}{32}$	$\frac{5wL^2}{192}$

Table D–1 *(Continued)*

	f_{1y}	m_1	Loading case	f_{2y}	m_2
8.	$\frac{-wL}{3}$	$\frac{-wL^2}{15}$	w (parabolic loading) 	$\frac{-wL}{3}$	$\frac{wL^2}{15}$
9.	$\frac{-M(a^2+b^2-4ab+L^2)}{L^3}$	$\frac{Mb(2a-b)}{L^2}$		$\frac{M(a^2+b^2-4ab+L^2)}{L^3}$	$\frac{Ma(2b-a)}{L^2}$



Positive nodal force conventions

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Table for Gauss points for integration from minus one to one $\int_{-1}^{1} y(x)dx = \sum_{i=1}^{n} W_i y_i$:

Number of Points	Locations, x_i	Associated Weights, W_i
1	x_1=0.000	2.000
2	x_1=+0.57735026918962 x_2=-0.57735026918962	1.000 1.000
3	x_1=+0.77459666924148 x_2=0.000 x_3=-0.77459666924148	5/9=0.555... 8/9=0.888... 5/9=0.555...
4	x_1=+0.8611363116 x_2=+0.3399810436 x_3=-0.3399810436 x_4=-0.8611363116	0.3478548451 0.6521451549 0.6521451549 0.3478548451