Prince of Songkla University
Faculty of Engineering

Midterm Test
6 August 2006
215-342 Mechanics of Fluids II

Semester 1/2549
13:30 – 16:30
Room R200

Name ______________________________ ID ______________

Direction:

1. All types of calculators, and dictionaries are permitted.
2. There are totally 5 problems, 8 pages. Solve all of them.
3. A two-page, hand-written A4 paper is allowed. No photocopy, please.

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Instructors

Problem No.	**Full score**	**Your mark**
1	**10**	
2	**10**	
3	**10**	
4	**10**	
5	**20**	
Total	**60**	

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215-342 – Mechanics of Fluids II
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1. The velocity of air in the diverging pipe shown in the figure is given by

$$V_1 = 4t^2 \quad \text{m/s}$$

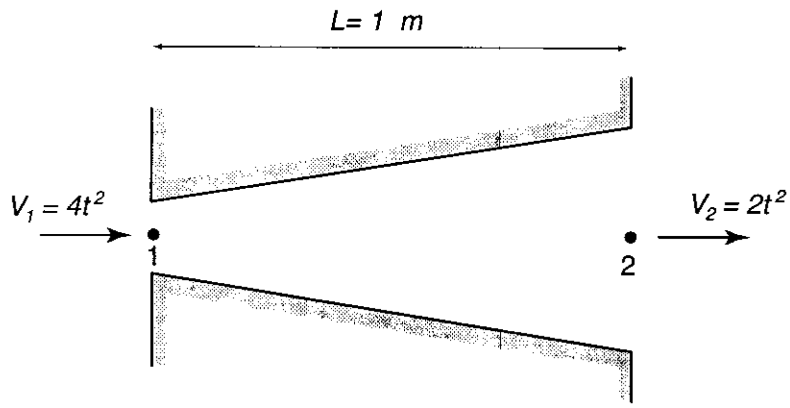
$$V_2 = 2t^2 \quad \text{m/s}$$

where t is in seconds.

(a) Determine local accelerations at point 1 and 2 after 10 seconds.

(b) Determine average convective acceleration between these two points after 10 seconds.

(10 points)



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2. The velocity field of a flow is given by

$$u = x^2 + y^2 + z^2$$
$$v = xy + yz + z^2$$
$$w = -3xz - z^2/2 + 2$$

(a) Determine the volumetric dilation rate, and interpret the results.

(b) Determine the rotation vector. Is the flow irrotational?

(10 points)

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3. The radial velocity component in an incompressible, two-dimensional flow field $(V_z = 0)$ is

$$V_r = 2r + 3r^2 \sin\theta$$

Determine the corresponding tangential velocity component, V_θ. (10 points)

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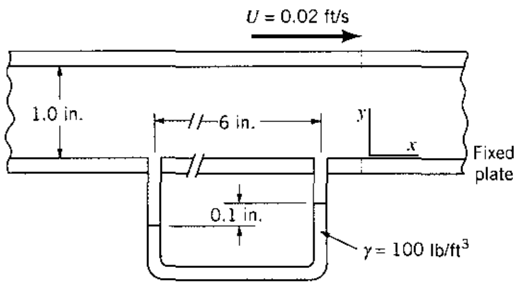
4. A viscous fluid (specific weight = 80 lb/ft^3; viscosity 0.03 lb.s/ft^2) is contained between two infinite, horizontal parallel plates as shown in the figure below. The fluid moves between the plates under the action of a pressure gradient, and the upper plate moves with a velocity U while the bottom plate is fixed. A U-tube manometer connected between two points along the bottom indicates a differential reading of 0.1 in. If the upper plate moves with a velocity of 0.02 f/s, at what distance from the bottom plate does the maximum velocity in the gap between the two plates occur? Assume laminar flow. (10 points)



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5. A viscous fluid is contained between two infinitely long, vertical, concentric cylinders. The outer cylinder has a radius r_o and rotates with an angular velocity ω. The inner cylinder is fixed and has a radius r_i. Make use of the Navier-Stokes equations to obtain an exact solution for the velocity distribution in the gap. Assume that the flow in the gap is axisymmetric (neither velocity nor pressure are functions of angular position θ within the gap) and that there are no velocity components other than the tangential component. The only body force is the weight. (20 points)

Life Savior

Volumetric Dilation

$$\frac{1}{\delta\forall}\frac{d(\delta\forall)}{dt}=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}$$

Rotation

$$\boldsymbol{\omega}=\frac{1}{2}\left(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}\right)\mathbf{i}+\frac{1}{2}\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)\mathbf{j}+\frac{1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)\mathbf{k}$$

Acceleration

$$a_x=\frac{\partial u}{\partial t}+\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)$$

$$a_y=\frac{\partial v}{\partial t}+\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)$$

$$a_z=\frac{\partial w}{\partial t}+\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)$$

Continuity Equation

Cartesian coordinates

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0$$

Cylindrical coordinates

$$\frac{\partial\rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r V_r)+\frac{1}{r}\frac{\partial}{\partial\theta}(\rho V_\theta)+\frac{\partial}{\partial z}(\rho V_z)=0$$

Navier-Stokes Equations

Cartesian coordinates

x-component: $$\rho g_x-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)=\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)$$

y-component: $$\rho g_y-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)=\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)$$

z-component: $$\rho g_z-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)=\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)$$

Cylindrical coordinates

r-component:

$$\rho g_r - \frac{\partial p}{\partial r} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_r}{\partial r}\right) - \frac{V_r}{r^2} + \frac{1}{r^2}\frac{\partial^2 V_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial V_\theta}{\partial \theta} + \frac{\partial^2 V_r}{\partial z^2}\right]$$
$$= \rho\left(\frac{\partial V_r}{\partial t} + V_r\frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta} - \frac{V_\theta^2}{r} + V_z\frac{\partial V_r}{\partial z}\right)$$

θ-component:

$$\rho g_\theta - \frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_\theta}{\partial r}\right) - \frac{V_\theta}{r^2} + \frac{1}{r^2}\frac{\partial^2 V_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial V_r}{\partial \theta} + \frac{\partial^2 V_\theta}{\partial z^2}\right]$$
$$= \rho\left(\frac{\partial V_\theta}{\partial t} + V_r\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{V_r V_\theta}{r} + V_z\frac{\partial V_\theta}{\partial z}\right)$$

z-component:

$$\rho g_z - \frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V_z}{\partial \theta^2} + \frac{\partial^2 V_z}{\partial z^2}\right]$$
$$= \rho\left(\frac{\partial V_z}{\partial t} + V_r\frac{\partial V_z}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_z}{\partial \theta} + V_z\frac{\partial V_z}{\partial z}\right)$$