มหาวิทยาลัยสงขลานครินทร์

คณะวิศวกรรมศาสตร์

สอบปลายภาค ประจำภาคการศึกษา 1 **ปีการศึกษา 2549**

วันที่ 10/10/06 **เวลา** 9.00-12.00

วิชา 220-501 Matrix Structural Analysis

ห้องสอบ A203

ชื่อ-สกุล................................ รหัส.............................

คำชี้แจง

1.ข้อสอบทั้งหมดมี 6 ข้อ คะแนนรวม 120 คะแนน ดังแสดงในตารางข้างล่าง

2.ข้อสอบมีทั้งหมด 4 หน้า (ไม่รวมปก) ผู้สอบต้องตรวจสอบว่ามีครบทุกหน้าหรือไม่ (ก่อนลงมือทำ) และห้ามแกะหรือฉีกข้อสอบออกจากเล่ม

3.ให้ทำหมดทุกข้อลงในสมุดคำตอบ

4.อนุญาตให้ใช้เครื่องคิดเลขได้ทุกชนิด

5.กระดาษทดที่แจกให้ไม่ต้องส่งคืน ถ้าไม่พอขอเพิ่มที่อาจารย์คุมสอบ

6.ห้ามหยิบ หรือยืมสิ่งของใดๆ ของผู้อื่นในห้องสอบ

7. One Page of Note Allowed

8. ***GOOD LUCK***

ตารางคะแนน

ข้อที่	คะแนนเต็ม	ได้
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
รวม	120	

Lecturer: Asst. Prof. Dr. Suchart Limkatanyu

Problem 1. (20 points)

For the uniform simply supported beam carrying a central point load as shown below, use the **Principle of Virtual Displacements** with the given assumed displacement as:

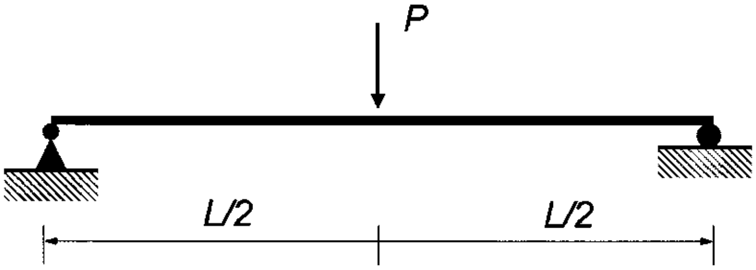
$$v(x) = a_1 \sin\left(\frac{\pi x}{L}\right) + a_2 \sin\left(\frac{3\pi x}{L}\right)$$

(1) Verify that this given displacement field is acceptable.

(2) Find the vertical displacement at mid-span (It should be larger or smaller that the exact value, give the supporting reason).

(3) Find the bending moment at the mid-span section.

(4) Find the reaction forces at the left and right ends.



Note:

$$\int_0^L \sin\left(\frac{m\pi x}{L}\right)\sin\left(\frac{n\pi x}{L}\right)dx = \frac{L}{2} : if\ m = n$$
$$= 0 : if\ m \neq n$$

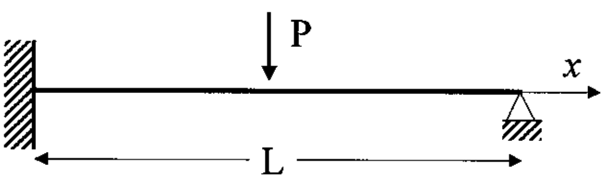
Problem 2. (20 points)

(1) In your own words, explain the difference between strong form and weak form of a beam problem.

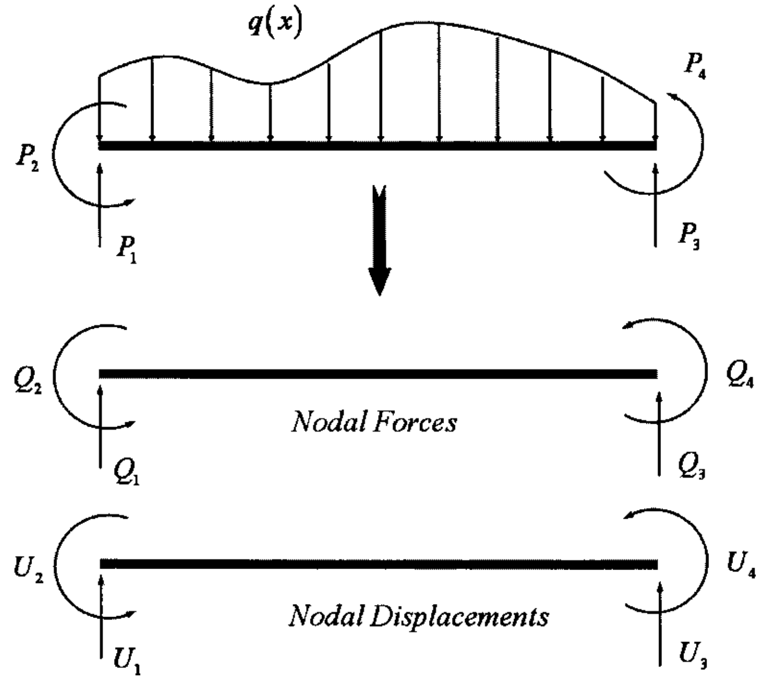
(2) In your own words, explain the difference between the Rayleigh-Ritz method and the displacement-based finite element method.

(3) Consider the beam shown below. Consider a Rayleigh-Ritz solution of the form $v(x) = c_1\ \phi(x)$. What are the conditions that $\phi(x)$ must satisfy?

Suggest an acceptable expression for $\phi(x)$.



Problem 3. (20 points)



For the beam subjected to the distributed load $q(x)$ and the end forces $\mathbf{P}$, show equivalence between principle of Virtual Displacements, and Principle of Stationary Potential Energy. In other words, show that both principles lead to the same equilibrium equation:

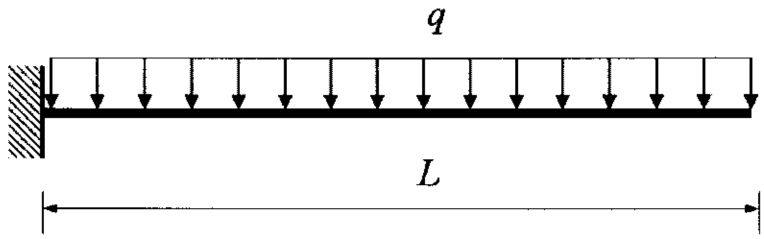
$$\mathbf{K}_{ele}\mathbf{U} = \mathbf{Q}$$

where

$$\mathbf{K}_{ele} = \int_L \mathbf{B}^T(x) IE(x) \mathbf{B}(x)\,dx$$

$$\mathbf{Q} = \mathbf{P} + \int_L \mathbf{N}^T q(x)\,dx$$

Problem 4: (20 points)



Given assumed displacement as:

$$v(x) = c_2 x^2 + c_3 x^3 + c_4 x^4$$

For the uniform cantilever beam carrying a uniformly distributed load as shown above, use the **Principle of Minimum Potential Energy** to show that the generalized stiffness equations are given as:

$$\frac{IE}{2}\begin{bmatrix} 8L & 12L^2 & 16L^3 \\ 12L^2 & 24L^3 & 36L^4 \\ 16L^3 & 36L^4 & \frac{288}{5}L^5 \end{bmatrix}\begin{Bmatrix} c_2 \\ c_3 \\ c_4 \end{Bmatrix} = \begin{Bmatrix} \frac{qL^3}{3} \\ \frac{qL^4}{4} \\ \frac{qL^5}{5} \end{Bmatrix}$$

Will this assumed displacement result in the exact solution?

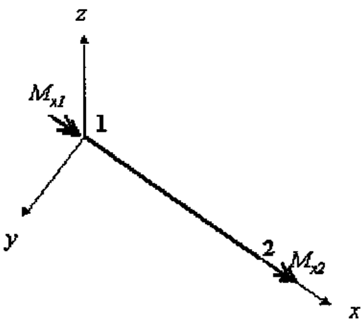
What will happen if you include the term $c_5 x^5$ into the assumed displacement field?

Problem 5: (20 points)

Consider the two-node element shown below. The *torsional response* of the element is of interest. The nodal displacements are $\mathbf{U} = \{\theta_{x1} \quad \theta_{x2}\}^T$, the corresponding nodal forces are $\mathbf{P} = \{M_{x1} \quad M_{x2}\}^T$. The cross section is circular and the radius changes according to $R(x) = R_0(1 - x/2L)$, where R_0 is the radius at x=0.

Consider a displacement-based formulation. Write down the expression for the element stiffness matrix, and specify the expressions for each term that appears in the stiffness matrix. *Do not solve the integrals!!!*

Do you expect any of the above formulations to be "exact"? Why?



Problem 6: (20 points)

Consider the 2-node beam element shown. It has 3 degrees of freedom per node: vertical displacement v_i, rotation θ_i and curvature κ_i (i=1,2). The vertical displacement field $v(x)$ is written in terms of the shape function $N_i(x)$ and the nodal displacements **d** as: $v(x) = \mathbf{N}(x)\,\mathbf{d} = \{N_1(x) \quad N_2(x) \quad N_3(x) \quad N_4(x) \quad N_5(x) \quad N_6(x)\}\ \{v_1 \quad \theta_1 \quad \kappa_1 \quad v_2 \quad \theta_2 \quad \kappa_2\}^T$

a) What is the order of the six polynomials $N_i(x)$, i=1,6?

b) Show the procedure you would follow to determine the interpolation functions (do not derive them!!)

c) What is the main drawback of the curvature continuity of this element?