PRINCE OF SONGKLA UNIVERSITY
FACULTY OF ENGINEERING

Final Examination: Semester I | Academic Year: 2006

Date: October 4, 2006 | Time: 9:00-12:00

Subject: 230-601 Advanced Engineering Mathematics for Chemical Engineers | Room: R200

อนุญาตให้นำเอกสารและเครื่องคำนวณทุกชนิดเข้าห้องสอบได้

ทุจริตในการสอบโทษขั้นต่ำคือปรับตกในรายวิชาที่ทุจริต
และพักการศึกษา 1 ภาคการศึกษา

Please do all 5 questions including bonus. Show all your work to receive full or partial credit. Total score is 150.

This exam has totally 4 pages.

Question #	Total Score	Score
1	20	
2	20	
3	40	
4	50	
5	20	
Total	150	

สุกฤทธิรา รัตนวิไล

1. $$x^2 \frac{d^2 y}{dx^2} + x\frac{dy}{dx} - x^2 y = 0$$

Define: $y = T - T_A$ and $X = \lambda r$,
T = Temperature, r = radius
T_A = ambient temperature, λ = constant

Boundary condition: $r = R_P \rightarrow T = T_P$
$r = R \rightarrow T = T_A$

Using Bessel Equation to solve for temperature as a function of r; **(20 points)**

2. Using Laplace Transform solve the differential equation. **(20 points)**

$$t\frac{d^2 y}{dt^2} - (1-t)\frac{dy}{dt} + \beta y = 0$$

Show the solution when $\beta = 1$ and $\beta = 2$

3. A semi-infinite insulated rod has an initial and boundary conditions as follow:
$T(x,0) = To$
$T(\infty,t) = To$
$T(0,t) = f(t)$

Use Newton's law develop the partial differential equation describing the temperature of the semi-infinite rod as the function of time (t) and position (x) and solved PDEs by using Laplace Transform. **(40 points)**

4. An incompressible fluid, initially at rest in a circular tube of length L, is subjected to the application of a step change in pressure gradient, so that the larminar conditions, the local velocity along the tube axis obeys

$$\rho\frac{\partial v}{\partial t} = \mu\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right) + \frac{\Delta p}{L}$$

At the final steady state, the velocity profile obeys

$$\mu\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \bar{v}}{\partial r}\right) + \frac{\Delta p}{L} = 0$$

Hence, since

$$r = R, \bar{v} = 0$$

$$r = 0, \frac{d\bar{v}}{dr} = 0$$

Then

$$\bar{v}(r) = \left(\frac{R^2 \Delta p}{4\mu L}\right)\left[1 - \left(\frac{r}{R}\right)^2\right]$$

As it stands, the equation describing transient velocity is inhomogeneous, and a separation of variables approach will fail. This can be remedied by the following technique. Define velocity as being made up of two parts, a steady state part plus a deviation from steady state

$$v(r,t) = \bar{v}(r) + y(r,t)$$

When this is inserted above, the steady part causes cancellation of $\frac{\Delta p}{L}$; hence,

$$\rho \frac{\partial y}{\partial t} = \mu \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial y}{\partial r}\right)$$

a. Show that the deviation velocity $y(r,t)$ must obey the initial condition **(20 points)**

$$y(r,0) = -2v_o\left[1 - \left(\frac{r}{R}\right)^2\right]$$

where

$$2v_o = \frac{R^2 \Delta p}{4\mu L}$$

V_0 is the average tube velocity at steady state; symmetry conditions are also obeyed; hence,

$$y(R,t) = 0 \text{ and } \frac{\partial y(0,t)}{\partial r} = 0$$

b. The equation and boundary conditions are now homogeneous; apply the separation variables method to find $y(\xi,\tau)$ where **(30 points)**;

$$\xi = \frac{r}{R}; \text{ and } \tau = \frac{\mu t}{\rho R^2}$$

5. Answer the following questions **in THAI** based on your understanding **(20 points)**, 5 points/each question.

   a. What is separation of variables method?
   b. What is Sturm-Liouville equation?
   c. Which is condition of PDEs that can apply Sturm-Liouville equation?
   d. What is the difference between inhomogeneous and homogeneous boundary?