PRINCE OF SONGKLA UNIVERSITY
FACULTY OF ENGINEERING

Midterm Examination: Semester II — Academic Year: 2006
Date: 23 December 2006 — Time: 9:00-12:00
Subject: 240-650 Principles of Pattern Recognition — Room: R200

Instructions:

This exam has 6 problems, 11 pages and 75 points. You may use the back of the pages for scratch work. This exam is open books and notes.

Name .... ID ....

“ทุจริตในการสอบ โทษขั้นต่ำ คือ พักการเรียน 1 ภาคการศึกษา และปรับตกในรายวิชาที่ทุจริต"

1. Find the eigenvalues and eigenvectors of matrix **A**, where $\mathbf{A} = \begin{bmatrix} 6 & 16 \\ -1 & -4 \end{bmatrix}$ (10 points)

2. Suppose x is a random variable whose probability density function is defined by

$$p(x) = \begin{cases} 1/(b-a) & a \le x \le b \\ 0 & elsewhere \end{cases}, \qquad \text{(where b > a)}$$

2.1 sketch $p(x)$ (1 point)
2.2 compute the mean of x (3 points)
2.3 compute the variance of x. (3 points)
2.4 If $p(y) = N(2,5)$, $a = 2$ and $b = 7$, determine if $p(x)$ is greater or less than $p(y)$ for $x = 3$ and $y = 3$? (3 points)

3. Box 1 contains 20 red balls and 10 blue balls. Box 2 contains 50 red balls and 2 blue balls. Box 3 contains 60 yellow balls and 10 red balls. If 2 boxes are randomly selected and all the balls from those two selected boxes are transferred to Box 4 which already contains 5 blue balls. Then a ball is randomly selected from Box 4. Answer the following questions:

3.1 What is the probability that the selected ball is red? (5 points)

3.2 If the selected ball from Box 4 is blue, what is the probability that the ball comes from Box 2? (5 points)

4. Suppose we have 3 classes of patterns whose features are in 2-D feature space. Each has the following underlying distributions:

$$p(\mathbf{x} \mid \omega_1) \cong N(\mathbf{0}, \mathbf{I})$$

$$p(\mathbf{x} \mid \omega_2) \cong N\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \mathbf{I}\right)$$

$$p(\mathbf{x} \mid \omega_3) \cong \frac{1}{1.5} N\left(\begin{bmatrix} -0.5 \\ 0.6 \end{bmatrix}, \mathbf{I}\right)$$

with $P(\omega_1) = 1/2$ and $P(\omega_2) = P(\omega_3) = 1/4$.

By explicit calculation of posterior probabilities, classify the point $x = \begin{bmatrix} 0 \\ 0.4 \end{bmatrix}$ for minimum probability of error. (15 points)

5. A random variable x is normally distributed with $\mu_x = 20$ and $\sigma_x = 4$. Let $y = 2x + 5$. Determine the mean and variance of y. (10 points)

6. Class 1 and Class 2 has the feature vectors as shown in the following table:

Class 1		Class 2	
x1	x2	x1	x2
-3	0	6	0
-2	1	2	0
-1	0	4	2
-2	-1	4	-2

The features of each class are assumed to have a Guassian distribution function (normally distributed).

6.1 Use the maximum likelihood parameter estimation method to determine $p(\mathbf{x}|\boldsymbol{\theta}_1)$ and $p(\mathbf{x}|\boldsymbol{\theta}_2)$ (10 points)

6.2 If $P(\omega_1) = 0.6$ and $P(\omega_2) = 0.4$, use the minimum error criterion to determine the decision function. (10 points)