ชื่อ...รหัส.......................................

มหาวิทยาลัยสงขลานครินทร์

คณะวิศวกรรมศาสตร์

การสอบกลางภาค ประจำภาคการศึกษาที่ 2 ประจำปีการศึกษา 2549

วันที่ 16 ธันวาคม 2549 เวลา 13.30-16.30

วิชา 215-612 ระเบียบวิธีไฟไนท์เอลิเมนท์ (Finite Element Method) ห้อง R200

คำสั่ง

1. ไม่อนุญาตให้นำหนังสือหรือเอกสารอื่นใดเข้าห้องสอบ
2. อนุญาตให้ใช้เครื่องคิดเลขได้ทุกรุ่น
3. ใช้ดินสอหรือปากกาทำข้อสอบก็ได้
4. ใช้เวลาทำ 3 ชั่วโมง
5. ข้อมูลเพิ่มเติม (Attachment) มีให้ในหน้าสุดท้ายของข้อสอบ

EXAM1:

ข้อสอบมีจำนวน 5 ข้อ ให้ทำทุกข้อ

ข้อ 1.________________ (20 คะแนน)

ข้อ 2.________________ (15 คะแนน)

ข้อ 3.________________ (20 คะแนน)

ข้อ 4.________________ (20 คะแนน)

ข้อ 5.________________ (25 คะแนน)

รวม__________________(100 คะแนน)

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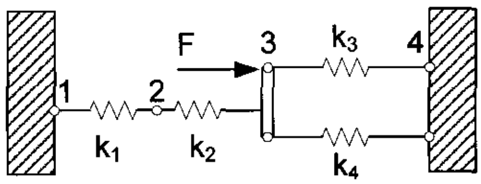
1. (20 points) The system below consists of four springs, of stiffness k_1, k_2, k_3 and k_4. Find the horizontal displacement of node 2 and node 3 for which the system is in equilibrium, using the displacement method.



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2. (15 points) In our classroom discussions on elements with axial degrees of freedom, we worked with 2-node linear elements, and discussed the use of higher order polynomials with additional nodal degrees of freedom at other positions in the element.

The general form for a displacement function is a polynomial of the form

$$U(x) = a_o + a_1 x + a_2 x^2 + a_3 x^3 + \ldots + a_n x^n$$

(a) How many terms are needed in a polynomial displacement function for a bar element with nodes (and only axial displacement degrees of freedom) at x=0, L/2, and L (u_1, u_2, u_3)? In other words, what polynomial should be used?

(b) What conditions can be used to solve for the coefficients a_i in terms of the nodal degrees of freedom u_i for this element?

(c) What do we mean when we say that a displacement function is "complete"?

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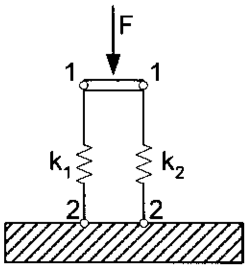
3. (20 points) The system below consists of two springs in parallel, of stiffness k_1 and k_2. Find the vertical displacement of node 1 for which the system is in equilibrium, using the method of minimum potential energy.



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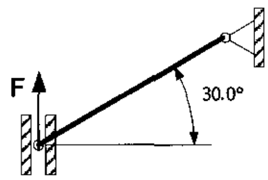
4. (20 points) A bar of constant cross-sectional area A, modulus E, length L, is oriented at 30 degrees to the horizontal. It is fixed at the right end. The left end is on a roller support such that the HORIZONTAL displacement is zero, but it can move freely in the VERTICAL direction. The left end has a force F applied in the vertical direction (upwards)

Find the <u>stress</u> in the bar by a finite element method.



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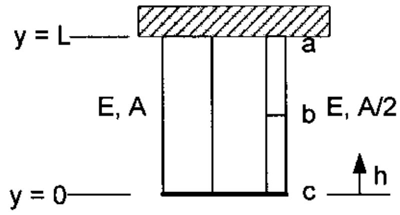
5. (25 points) The bar assemblage shown below can be solved for deflection along the vertical axis, by the direct stiffness method. The left member is of area A, and the right one of A/2.



[5-1] Assume the upper end "a" is fixed at zero displacement, and the lower end "c" is moved upward by an amount "h". Set up a global stiffness equation for the system in terms of displacements u_a, u_b, u_c, and show how to apply boundary conditions and load to the system of equations.

[5-2] Solve for the displacement of the point "b" in the center of the right member.

[5-3] Calculate how much force is in the left member and in the right member.

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ATTACHMENT

→ 1D Spring

Stiffness Matrix $[K]=\begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \quad \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$

→ 2D Bar (along x-axis) axial deflection

Stiffness Matrix $[K]=\frac{AE}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} d_{1x} \\ d_{1y} \\ d_{2x} \\ d_{2y} \end{bmatrix}$

→ 2D Bar (at angle θ to x-axis)

$C = \cos\theta$

$S = \sin\theta$

Stiffness Matrix $[K]=\frac{AE}{L}\begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix} \quad \begin{bmatrix} d_{1x} \\ d_{1y} \\ d_{2x} \\ d_{2y} \end{bmatrix}$

→ Transformation of a vector from global d to local $\hat{d}$

(Local x-axis rotated counterclockwise by angle θ from global x-axis)

$$\begin{bmatrix} \hat{d}_x \\ \hat{d}_y \end{bmatrix}=\begin{bmatrix} C & S \\ -S & C \end{bmatrix}\begin{bmatrix} d_x \\ d_y \end{bmatrix}$$

Stress in a bar at angle θ to global axis:

$\sigma = C'd_{global} \qquad C'=\frac{E}{L}[-1 \quad 1]\begin{bmatrix} C & S & 0 & 0 \\ 0 & 0 & C & S \end{bmatrix}$

Bar element with linear displacement function

$N_1 = 1-\frac{x}{L} \qquad N_2 = \frac{x}{L} \qquad u(x)=[N_1 \quad N_2]\begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$