Name.....Student I.D.....

Department of Mining and Materials Engineering

Faculty of Engineering

Prince of Songkla University

Mid-term Examination for Semester: 2 Academic Year: 2006

Date: December 19, 2006 Time: 9.00-12.00

Subject: 237-508 Structures and Mechanical Properties of Materials Room: R200

Instruction

1. There are 3 problem sets. Please do all of them. Write your answers in the space provided. If you need more space, you can write on the back of paper.
2. Only one (1) piece of A4-size note is allowed. You may write on both sides of the note. Please return it with your answers.
3. Dictionary, calculator, and stationery are also allowed.
4. Text books and other studying materials are not allowed.
5. This mid-term exam is counted for 30% of the total grade.

Asst. Prof. Dr. Thawatchai Plookphol

Problem No.	Full Score (points)	Student's Score (points)
1.	60	
2.	40	
3.	50	
Total	150	

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1. The 3-D state of stress is given by:

$$\sigma_{ij} = \begin{bmatrix} 0 & -240 & 0 \\ -240 & 200 & 0 \\ 0 & 0 & -280 \end{bmatrix} \text{ MPa}$$

(a) Calculate the three invariants of stress (I_1, I_2, I_3) (15 points)

(b) Calculate the principal stresses $(\sigma_1, \sigma_2, \sigma_3)$**.** Please show your work. (30 points)

(c) Write a new stress tensor σ'_{ij} from the principal stresses (part (b)) in the form given below, using the convention that $\sigma_1 > \sigma_2 > \sigma_3$. **Calculate the invariants of the stress tensor** σ'_{ij} **and compare with the answer in part (a).** (15 points)

$$\sigma'_{ij} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} \text{ MPa}$$

Given:

$$det\begin{bmatrix} \sigma_{11}-\sigma & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22}-\sigma & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33}-\sigma \end{bmatrix} = 0$$

$$I_3 - \sigma I_2 + \sigma^2 I_1 - \sigma^3 = 0$$

where,

$$I_1 = \sigma_{11} + \sigma_{22} + \sigma_{33}$$

$$I_2 = (\sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21}) + (\sigma_{22}\sigma_{33} - \sigma_{23}\sigma_{32}) + (\sigma_{11}\sigma_{33} - \sigma_{13}\sigma_{31})$$

$$I_3 = \sigma_{11}(\sigma_{22}\sigma_{33} - \sigma_{23}\sigma_{32}) + \sigma_{21}(\sigma_{13}\sigma_{32} - \sigma_{12}\sigma_{33}) + \sigma_{31}(\sigma_{12}\sigma_{23} - \sigma_{13}\sigma_{22})$$

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2. The three-dimensional state of stress given in Problem 1:

$$\sigma_{ij} = \begin{bmatrix} 0 & -240 & 0 \\ -240 & 200 & 0 \\ 0 & 0 & -280 \end{bmatrix} \text{ MPa}$$

Determine the normal stress (σ) **and shear stress** (τ) acting on a plane with direction cosines

$$n_1 = \frac{1}{\sqrt{3}} \qquad n_2 = \frac{1}{\sqrt{3}} \qquad n_3 = \frac{1}{\sqrt{3}}$$

(40 points)

Given:

$$\overline{S} = \overline{\overline{\sigma}} \cdot \hat{n} \qquad \text{or} \qquad S_i = \sum_{j=1}^{3} \sigma_{ij} n_j$$

$$S^2 = S_1^2 + S_2^2 + S_3^2$$

$$\sigma = S_1 \cdot n_1 + S_2 \cdot n_2 + S_3 \cdot n_3$$

$$S^2 = \sigma^2 + \tau^2$$

where,

S = Total stress acting on the plane,
σ = Normal stress acting on the plane,
τ = Shear stress acting on the plane,
n_1, n_2, n_3 are direction cosines.

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3. An aluminum single crystal (cubic) experiences the state of stress of

$$\sigma_{ij} = \begin{bmatrix} 0 & -240 & 0 \\ -240 & 200 & 0 \\ 0 & 0 & -280 \end{bmatrix} \text{ MPa}$$

Some elastic constants of aluminum are given by:

$$E_{11} = 63.7 \text{ GPa}, \qquad G_{12} = 28.5 \text{ GPa}, \qquad \nu_{12} = 0.31$$

(a) Determine the compliance matrix $[S]$ for the aluminum single crystal (i.e. compliance tensor) (20 points)

(b) Determine the engineering strain $[\varepsilon]$ in the aluminum single crystal by assuming deformation is linear elastic. (30 points)

Given: $S_{11} = \frac{1}{E_{11}}, \qquad S_{12} = -\frac{\nu_{12}}{E_{11}}, \qquad S_{44} = \frac{1}{G_{12}}$

$$[\varepsilon] = [S][\sigma] \qquad \text{or}$$

$$\varepsilon_{ij} = S_{ijkl}\sigma_{kl}$$

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