มหาวิทยาลัยสงขลานครินทร์

คณะวิศวกรรมศาสตร์

สอบปลายภาค ประจำภาคการศึกษา 2 ปีการศึกษา 2549

วันที่ 26 กุมภาพันธ์ 2550 เวลา 13.30 – 16.30.

วิชา CE 220-504: Introduction to Finite Element Method ห้องสอบ A 400

ชื่อ-สกุล..

รหัส...........................

คำชี้แจง

1.ข้อสอบทั้งหมดมี 6 ข้อ คะแนนรวม 100 คะแนน ดังแสดงในตารางข้างล่าง

2.ข้อสอบมีทั้งหมด 5 หน้า (รวมปก) ผู้สอบต้องตรวจสอบว่ามีครบทุกหน้าหรือไม่ (ก่อนลงมือทำ)

3.ให้ทำหมดทุกข้อลงในสมุดคำตอบ

4.อนุญาตให้ใช้เครื่องคิดเลขได้ทุกชนิด

5.ห้ามหยิบ หรือยืมสิ่งของใดๆ ของผู้อื่นในห้องสอบ

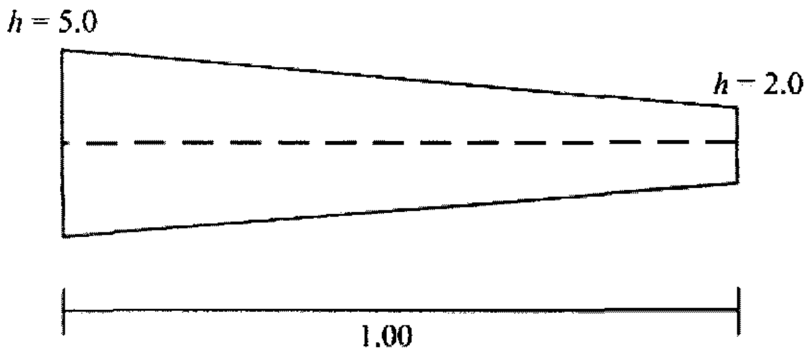
6. **Open Books**

ตารางคะแนน

ข้อที่	คะแนนเต็ม	ได้
1	10	
2	20	
3	25	
4	5	
5	15	
6	25	
รวม	100	

Problem 1 (10 Points)

To determine the element stiffness matrix of a tapered beam,



whose width is constant $b = 1$, but depth h varies linearly over the element as shown in the figure.

We have to integrate

$$\mathbf{K} = \int_0^1 B^2(x) EI(x) dx$$

where $B(x) = 6 - 12x$.

Use Gauss quadrature to integrate this problem by using only the minimum n-point that can get the exact result.

Problem 2 (20 Points)

In the finite element formulation of near incompressible isotropic materials (as well as plasticity and viscoelasticity) it is convenient to use the so-called *Lame constants* λ and μ instead of E and ν in the constitutive equations. Both λ and μ have the physical dimension of stress and are related to E and ν by

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}, \quad \mu = G = \frac{E}{2(1+\nu)}.$$

Conversely

$$E = \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu}, \quad \nu = \frac{\lambda}{2(\lambda + \mu)}.$$

Substitute the second equation into the following equation

$$\text{plane stress:} \quad \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ 2e_{xy} \end{bmatrix},$$

$$\text{plane strain:} \quad \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & 0 \\ \frac{\nu}{1-\nu} & 1 & 0 \\ 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ 2e_{xy} \end{bmatrix}.$$

to express the two stress-strain matrices in terms of λ and μ. Then split the stress-strain matrix **E** of plane strain as

$$\mathbf{E}=\mathbf{E}_{\mu}+\mathbf{E}_{\lambda}$$

in which $\mathbf{E}_{\mu}$ and $\mathbf{E}_{\lambda}$ contain only μ and λ, respectively, with $\mathbf{E}_{\mu}$ diagonal and $E_{\lambda 33}=0$. This is the Lame or $\{\lambda\ ,\ \mu\}$ splitting of the plane strain constitutive equations, which leads to the so-called **B**-bar formulation of near-incompressible finite elements. Express $\mathbf{E}_{\mu}$ and $\mathbf{E}_{\lambda}$ also in terms of E and v.

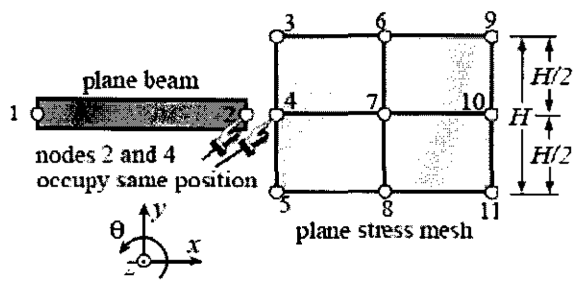
For the plane stress case perform a similar splitting in which where $\mathbf{E}_{\lambda}$ contains only $\bar{\lambda}=2\lambda\mu/(\lambda+2\mu)$ with $E_{\lambda 33}$ = 0, and $\mathbf{E}_{\mu}$ is a diagonal matrix function of μ and $\bar{\lambda}$. Express $\mathbf{E}_{\mu}$ and $\mathbf{E}_{\lambda}$ also in terms of E and v.

Problem 3 (25 Points)

A cantiveler beam-column is joined to a plane stress plate mesh as depicted in the figure. Both pieces move in the plane $\{x, y\}$. Plane stress elements have two degrees of freedom per node: two translations u_x and u_y along x and y, respectively, whereas a beam-column element has three: two translations u_x and u_y along x and y, and one rotation (positive CCW) θ_z about z. To connect the cantilever beam to the mesh, the following "gluing" conditions are applied:

(1) The horizontal (u_x) and vertical (u_y) displacements of the beam at their common node (2 of beam, 4 of plate) are the same.

(2) The beam end rotation θ_2 and the mean rotation of the plate edge 3–5 are the same. For infinitesimal displacements and rotations the latter is $\theta_{35}^{avg}=(u_{x5}-u_{x3})/H$.



Questions:

(a) Write down the three MFC conditions: two from (1) and one from (2), and state whether they are linear and homogeneous.

(b) Where does the above expression of θ_{35}^{avg} come from? (Geometric interpretation is ok.) Can it be made more accuracy by including u_{x4}? (To answer this question, observe that the displacements along 3-4 and 4-5 vary linearly. Thus the angle of rotation about z is constant for each of them and (for infinitesimal displacements) may be set equal to the tangent.)

(c) Write down the master-slave transformation matrix if $\{u_{x2}, u_{y2}, \theta_2\}$ are picked as slaves. It is sufficient to write down the transformation for the DOFs of nodes 2,3,4 and 5, which give a **T** of order 9×6, since the transformations for the other freedoms are trivial.

(d) If the penalty method is used, write down the stiffness equations of the three penalty elements assuming the same weight w is used. Their stiffness matrices are of order 2 × 2, 2 × 2 and 3 × 3, respectively. (Do not proceed further)

(e) If Lagrange multiplier adjunction is used, how many Lagrange multipliers will you need to append? (Do not proceed further)

Problems 4 (5 Points)

The free-free stiffness equations of a superelement are

$$\begin{bmatrix} 88 & -44 & -44 & 0 \\ -44 & 132 & -44 & -44 \\ -44 & -44 & 176 & -44 \\ 0 & -44 & -44 & 220 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 15 \\ 20 \end{bmatrix}$$

Eliminate u_2 and u_3 from the above equation by static condensation (using the explicit inverse formulas), and show (but do not solve) the condense equation system.

Problem 5 (15 Points)

Compute the entries of $\mathbf{K}^e$ for the following plane stress triangle:

$$x_1 = 0,\ y_1 = 0,\ x_2 = 3,\ y_2 = 1,\ x_3 = 2,\ y_3 = 2,$$

$$\mathbf{E} = \begin{bmatrix} 100 & 25 & 0 \\ 25 & 100 & 0 \\ 0 & 0 & 50 \end{bmatrix}, \qquad h = 1.$$

Problem 6 (25 Points)

Consider a tapered bar, shown in the following figure, subjected to its own weight.

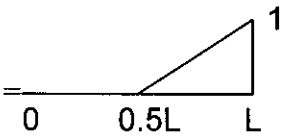
(a) Write the expression for the total potential energy of this bar.

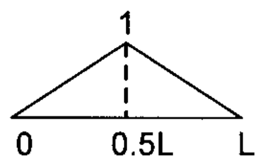
(b) From the principle of minimum total potential energy, obtain the governing equation (Euler-Largrange equation), and the boundary conditions of this bar. Also indicate the type of boundary conditions.

(c) Derive the analytical solution of this problem.

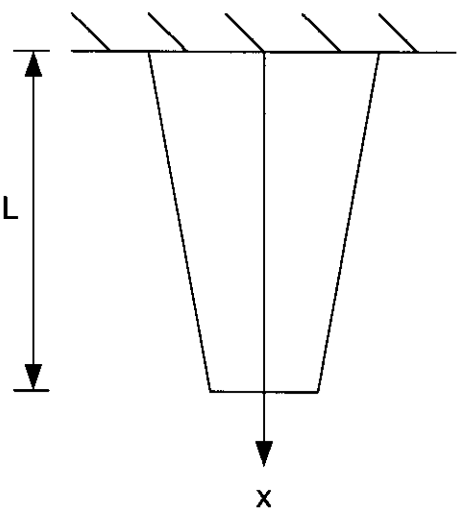
Let $\gamma = 10^4$ T/m^3; $E = 2\times10^7$ T/m^2; $L = 4$ m.

(d) If $A_0 = A_L = 30\ \text{cm}^2$, obtain the Rayleigh-Ritz solution by assuming $\hat{u} = a_1\mathrm{N}_1 + a_2\mathrm{N}_2$ where

$\mathrm{N}_1(x) =$ 

;$\mathrm{N}_2(x) =$ 

Determine also an axial force, $\hat{F}(x) = EA\dfrac{d\hat{u}}{dx}$.



γ = unit weight

E = Modulus of Elasticity

$A(0) = A_0$

$A(L) = A_L$

$A(x) = A_0 - \frac{x}{L}(A_0 - A_L)$