ชื่อ...รหัส.......................................

มหาวิทยาลัยสงขลานครินทร์

คณะวิศวกรรมศาสตร์

การสอบปลายภาค ประจำภาคการศึกษาที่ 2 ปีการศึกษา 2549

วันที่ 2 มีนาคม 2550 เวลา 13.30-16.30 น.

วิชา 215-612 ระเบียบวิธีไฟไนท์เอลิเมนท์ (Finite Element Method) ห้อง R300

คำสั่ง

1. ไม่อนุญาตให้นำหนังสือหรือเอกสารอื่นใดเข้าห้องสอบ
2. อนุญาตให้ใช้เครื่องคิดเลขได้ทุกรุ่น
3. ใช้ดินสอหรือปากกาทำข้อสอบก็ได้
4. ใช้เวลาทำ 3 ชั่วโมง

ทุจริตในการสอบ โทษต่ำสุด คือ พักการเรียน 1 ภาคการศึกษา และปรับตกในรายวิชาที่ทุจริต

*** มีข้อมูลเพิ่มเติมอยู่ด้านหลังของข้อสอบ ***

FINAL EXAM:

ข้อสอบมีจำนวน 5 ข้อ ให้ทำทุกข้อ

ข้อ 1.________________ (20 คะแนน)

ข้อ 2.________________ (30 คะแนน)

ข้อ 3.________________ (30 คะแนน)

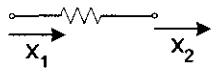
ข้อ 4.________________ (20 คะแนน)

ข้อ 5.________________ (20 คะแนน)

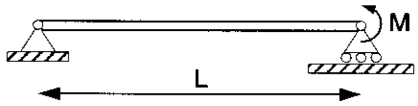
รวม________________(120 คะแนน)

ชื่อ...รหัส......................................

1. (20 points)
During the finite element course we developed several elements. Fill out the table below for these elements.

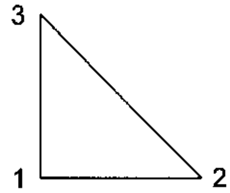
ELEMENT	List of Degrees of Freedom	Displacement function f(x) or f(x,y)	Sketch of Element (Label DOF)
"For example" Spring in 1D	x_1, x_2	$f(x)=a_1+a_2x$	
BAR in 2D space at angle to x-axis			
Beam along x-axis			
Frame at angle to x-axis			
2D Solid-CST			
2D Solid-LST			

2. (30 points)
For a **PINNED-GUIDED** beam of length L, with a moment on the Right End, (at x=L), use the finite element formulation to develop the matrix representation and solve for the **equation of deflection as a function of x in the beam**.

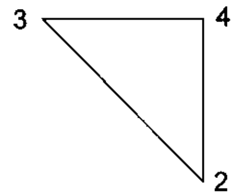


ชื่อ...รหัส......................................

3. (30 points) The stiffness matrices for two CST elements are given in terms of constants a, b, c, … and A, B, C, … etc. These elements are to be assembled into a square block 1-2-4-3 (x-axis is horizontal)



$$[K]^{(1)} = \begin{bmatrix} a & b & -c & -d & -e & -g \\ b & f & -g & -h & -d & -i \\ -c & -g & c & 0 & 0 & g \\ -d & -h & 0 & h & d & 0 \\ -e & -d & 0 & g & e & 0 \\ -g & -i & g & 0 & 0 & i \end{bmatrix} \quad \begin{Bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ x_3 \\ y_3 \end{Bmatrix}$$



$$[K]^{(2)} = \begin{bmatrix} A & B & -C & -D & -E & -G \\ B & F & -G & -H & -D & -I \\ -C & -G & C & 0 & 0 & G \\ -D & -H & 0 & H & D & 0 \\ -E & -D & 0 & G & E & 0 \\ -G & -I & G & 0 & 0 & I \end{bmatrix} \quad \begin{Bmatrix} x_2 \\ y_2 \\ x_3 \\ y_3 \\ x_4 \\ y_4 \end{Bmatrix}$$

What are the force components on node 3 resulting from a unit displacement, in the x direction, of node 2 of the assembled structure, with all other degrees of freedom fixed to zero?

ชื่อ..รหัส......................................

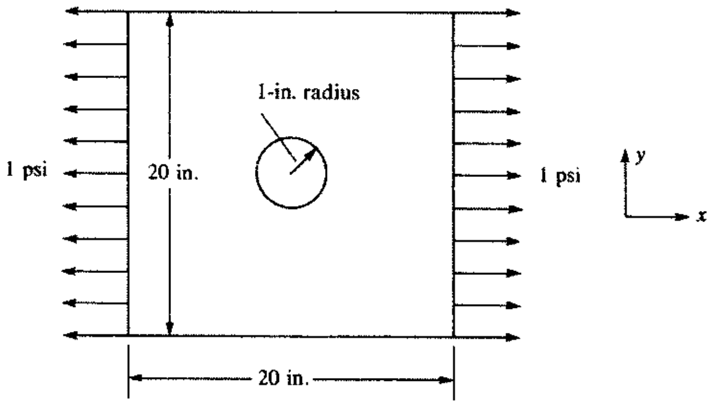
4. (20 points)
Describe the concepts of **plane stress** and **plane strain** in detail.

a) Plane Stress (10 points)

b) Plane Strain (10 points)

ชื่อ...รหัส...................................

5. (20 points) Explain and discuss in detail "how to model" the following problem by using finite element software such as MSC.Patran/Nastran. (Topics of discussion in assumption, element, geometry, material properties, mesh, BCs, load, symmetry, preprocessing, solver, post processing, etc.)



ชื่อ...รหัส..

ชื่อ..รหัส....................................

ATTACHMENT

→ 2D SPAR (along x-axis) axial deflection

$$\text{Stiffness Matrix } [K]=\frac{AE}{L}\begin{bmatrix}1 & 0 & -1 & 0\\ 0 & 0 & 0 & 0\\ -1 & 0 & 1 & 0\\ 0 & 0 & 0 & 0\end{bmatrix} \qquad \begin{bmatrix}d_{1x}\\ d_{1y}\\ d_{2x}\\ d_{2y}\end{bmatrix}$$

→ 2D Bar (at angle θ to x-axis)
C = cosθ
S = sinθ

$$\text{Stiffness Matrix } [K]=\frac{AE}{L}\begin{bmatrix}C^2 & CS & -C^2 & -CS\\ CS & S^2 & -CS & -S^2\\ -C^2 & -CS & C^2 & CS\\ -CS & -S^2 & CS & S^2\end{bmatrix} \qquad \begin{bmatrix}d_{1x}\\ d_{1y}\\ d_{2x}\\ d_{2y}\end{bmatrix}$$

→ Transformation of a vector from global d to local $\hat{d}$

(Local x-axis rotated counterclockwise by angle θ from global x-axis)

$$\begin{bmatrix}\hat{d}_x\\ \hat{d}_y\end{bmatrix}=\begin{bmatrix}C & S\\ -S & C\end{bmatrix}\begin{bmatrix}d_x\\ d_y\end{bmatrix}$$

→ 2D Beam (lateral deflection)

$$\text{Stiffness Matrix } [K]=\frac{EI}{L^3}\begin{bmatrix}12 & 6L & -12 & 6L\\ 6L & 4L^2 & -6L & 2L^2\\ -12 & -6L & 12 & -6L\\ 6L & 2L^2 & -6L & 4L^2\end{bmatrix} \qquad \begin{bmatrix}d_{1y}\\ \phi_1\\ d_{2y}\\ \phi_2\end{bmatrix}$$

Shape Function of beam element of length L in bending

$$\begin{aligned}v(x) &= [(2/L^3)(d_{1y}-d_{2y})+(1/L^2)(\phi_1+\phi_2)]x^3\\ &\quad +[-(3/L^2)(d_{1y}-d_{2y})-(1/L)(2\phi_1+\phi_2)]x^2\\ &\quad +\phi_1 x+d_{1y}\end{aligned}$$

ชื่อ...รหัส..

→ Stiffness Matrix for Frame Element at angle θ to x-axis is

$$[K]=\frac{E}{L}\begin{bmatrix} AC^2+DS^2 & [A-D]CS & -6IS/L & -[AC^2+DS^2] & -[A-D]CS & -6IS/L \\ & AS^2+DC^2 & 6IC/L & -[A-D]CS & -[AS^2+DC^2] & 6IC/L \\ & & 4I & 6IS/L & -6IC/L & 2I \\ & & & AC^2+DS^2 & [A-D]CS & 6IS/L \\ & \text{Symmetry} & & & AS^2+DC^2 & -6IC/L \\ & & & & & 4I \end{bmatrix} \begin{bmatrix} d_{1x} \\ d_{1y} \\ \phi_1 \\ d_{2x} \\ d_{2y} \\ \phi_2 \end{bmatrix}$$

where $D=12I/L^2$ $C=\cos\theta$ $S=\sin\theta$

work-equivalent forces

$$\int w(x)v(x)dx = m_1\phi_1 + m_2\phi_2 + f_{1y}d_{1y} + f_{2y}d_{2y}$$

****

Types of Elements:

TYPE	DOF	REAL CONTANTS
2D BAR	UX, UY	A
2D BEAM	UX, UY, ROTZ	A, I, TK
3D BEAM	UX, UY, UZ, ROTX, ROTY, ROTZ	A, IZ, IY, IZ, TKY, TKZ
3D BAR	UX, UY, UZ	A
3D PIPE	UX, UY, UZ, ROTX, ROTY, ROTZ	OD, TK
2D SOLID (PLANE STRESS, PLANE STRAIN, AXISSYMMETRIC)	UX, UY	
3D SOLID	UX, UY, UZ	
3D PLATE/SHELL	UX, UY, UZ, ROTX, ROTY, ROTZ	TK

****

Pascal triangle

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & x & & y & & \\ & x^2 & & xy & & y^2 & \\ x^3 & & x^2y & & xy^2 & & y^3 \end{array}$$

****

Table D-1 Equivalent joint forces f_0 for different types of loads

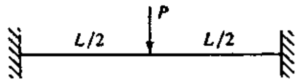
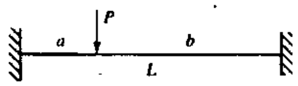
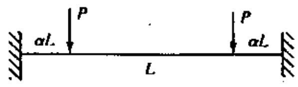
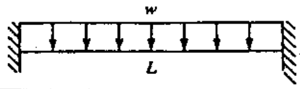
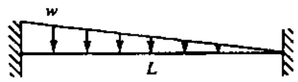
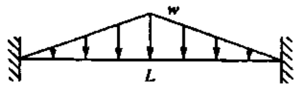
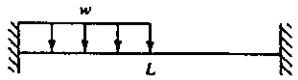
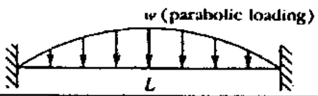
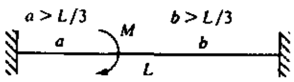
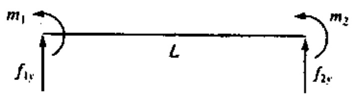
	f_{1y}	m_1	Loading case	f_{2y}	m_2
1.	$\frac{-P}{2}$	$\frac{-PL}{8}$		$\frac{-P}{2}$	$\frac{PL}{8}$
2.	$\frac{-Pb^2(L+2a)}{L^3}$	$\frac{-Pab^2}{L^2}$		$\frac{-Pa^2(L+2b)}{L^3}$	$\frac{Pa^2b}{L^2}$
3.	$-P$	$-\alpha(1-\alpha)PL$		$-P$	$\alpha(1-\alpha)PL$
4.	$\frac{-wL}{2}$	$\frac{-wL^2}{12}$		$\frac{-wL}{2}$	$\frac{wL^2}{12}$
5.	$\frac{-7wL}{20}$	$\frac{-wL^2}{20}$		$\frac{-3wL}{20}$	$\frac{wL^2}{30}$
6.	$\frac{-wL}{4}$	$\frac{-5wL^2}{96}$		$\frac{-wL}{4}$	$\frac{5wL^2}{96}$
7.	$\frac{-13wL}{32}$	$\frac{-11wL^2}{192}$		$\frac{-3wL}{32}$	$\frac{5wL^2}{192}$

Table D-1 *(Continued)*

	f_{1y}	m_1	Loading case	f_{2y}	m_2
8.	$\frac{-wL}{3}$	$\frac{-wL^2}{15}$		$\frac{-wL}{3}$	$\frac{wL^2}{15}$
9.	$\frac{-M(a^2+b^2-4ab+L^2)}{L^3}$	$\frac{Mb(2a-b)}{L^2}$		$\frac{M(a^2+b^2-4ab+L^2)}{L^3}$	$\frac{Ma(2b-a)}{L^2}$



Positive nodal force conventions

ชื่อ...รหัส......................................

Table for Gauss points for integration from minus one to one $\int_{-1}^{1} y(x)dx = \sum_{i=1}^{n} W_i y_i$:

Number of Points	Locations, x_i	Associated Weights, W_i
1	x_1=0.000	2.000
2	x_1=+0.57735026918962 x_2=-0.57735026918962	1.000 1.000
3	x_1=+0.77459666924148 x_2=0.000 x_3=-0.77459666924148	5/9=0.555... 8/9=0.888... 5/9=0.555...
4	x_1=+0.8611363116 x_2=+0.3399810436 x_3=-0.3399810436 x_4=-0.8611363116	0.3478548451 0.6521451549 0.6521451549 0.3478548451