

Name _____ Student I.D. _____

Prince of Songkla University
Faculty of Engineering

Midterm Test

30 July 2007

215-643 Heat Convection

Semester 1/2550

13:30 – 16:30

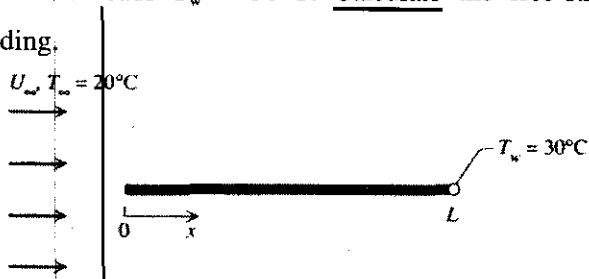
Room A205

Directions: There are 2 problems. Open book, open notes. Work all problems.

Pb. No.	Full score	Your mark
1	10	
2	30	
Total	40	

Kittinan Maliwan
Instructor

1. It is proposed to estimate the uniform velocity U_∞ of a stream of air of temperature 20°C by measuring the temperature of a thin metallic blade that is heated and inserted parallel to U_∞ in the airstream. The width of the blade (i.e., the dimension aligned with U_∞) is $L = 2$ cm. The blade is considerably longer in the direction normal to the figure; therefore, the boundary layer flow that develops is two-dimensional. The blade is heated volumetrically by an electric current so that 0.03 W electrical power is dissipated in each square centimeter of metallic blade. It is assumed that the blade is so thin that the effect of heat conduction through the blade (in the x direction) is negligible. A temperature sensor mounted on the trailing edge of the blade reads $T_w = 30^\circ\text{C}$. Calculate the free-stream velocity U_∞ that corresponds to this reading. (10 points)



2. Consider the laminar boundary layer *frictional heating* of an adiabatic wall parallel to a free stream (U_∞, T_∞). Modeling the flow as one with temperature-independent properties and assuming that the Blasius velocity solution holds, use scaling arguments to show that the relevant boundary layer energy equation for this problem is

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2$$

and that the wall temperature rise scales as U_∞^2/c_p when $Pr > 1$. Assuming that the wall is insulated ($\partial T/\partial y = 0$ at $y = 0$) and that $T \rightarrow T_\infty$ as $y \rightarrow \infty$. The similarity solution for the dimensionless temperature profile is

$$\theta(\eta) = \frac{T - T_\infty}{U_\infty^2/(2c_p)}$$

Show that the energy equation reduces to

$$\theta'' + \frac{Pr}{2} f \theta' + 2Pr(f'')^2 = 0$$

where $f(\eta)$ is the Blasius solution. Solving this equation subject to $\theta'(0) = 0$ and $\theta(\infty) = 0$, prove that the temperature rise in the boundary layer is

$$\theta(\eta) = 2Pr \int_\eta^\infty \left\{ \int_0^p [f''(\beta)]^2 \exp\left(\frac{Pr}{2} \int_0^\beta f(\gamma) d\gamma\right) d\beta \right\} \times \exp\left(-\frac{Pr}{2} \int_0^p f(m) dm\right) dp$$

(30 points)