PRINCE OF SONGKLA UNIVERSITY
FACULTY OF ENGINEERING

Midterm Examination: Semester II — Academic Year: 2007
Date: 5 January 2008 — Time: 13:30-16:30
Subject: 240-650 Principles of Pattern Recognition — Room: R201

Instructions:

This exam has 5 problems, 12 pages and 77 points. You may use the back of the pages for scratch work. This exam is <u>open books</u> and <u>notes</u>. It is allowed to use a calculator.

Name .... ID ....

"ทุจริตในการสอบ โทษขั้นต่ำ คือ พักการเรียน 1 ภาคการศึกษา และปรับตกในรายวิชาที่ทุจริต"

1. Find the eigenvalues and eigenvectors of matrix **A**, where $\mathbf{A} = \begin{bmatrix} 5 & -2 \\ 5 & 1 \end{bmatrix}$ (10 poin s)

2. Suppose x and y are random variables whose probability density functions are defir ed by

$$p(x) = \begin{cases} 0.1 & 0 \leq x \leq 10 \\ 0 & elsewhere \end{cases}, \qquad \text{and}$$

$$p(y) = \begin{cases} y & 0 \leq y \leq 1 \\ 1 - y & 1 < y \leq 2 \\ 0 & elsewhere \end{cases}$$

2.1 sketch $p(x)$ and $p(y)$ (2 point)
2.2 compute the mean of x, and y (6 points)
2.3 compute the variance of x and y (6 points)
2.4 determine if $p(x)$ is greater or less than $p(y)$, if $x = 0.75$ and $y = 0.5$? (3 points)

3. Suppose we have 3 classes of patterns whose features are in 2-D feature space. Each has the following underlying distributions:

$$p(\mathbf{x} \mid \omega_1) \cong N(\mathbf{0}, \mathbf{I})$$

$$p(\mathbf{x} \mid \omega_2) \cong N\left(\begin{bmatrix} 1 \\ 1.5 \end{bmatrix}, \mathbf{I}\right)$$

$$p(\mathbf{x} \mid \omega_3) \cong 0.5N\left(\begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix}, \mathbf{I}\right)$$

with $P(\omega_1) = 0.2$, $P(\omega_2) = 0.5$, and $P(\omega_3) = 0.3$

By explicit calculation of posterior probabilities, classify the point $x = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}$ for minimum probability of error. (15 points)

4. Class 1 and Class 2 has the feature vectors as shown in the following table:

Class 1		Class 2	
x1	x2	x1	x2
-3	0	4	0
-1	1	2	0
-1	0	3	2
-2	0	4	-2

The features of each class are assumed to have a Gaussian distribution function (norm:lly distributed).

6.1 Use the maximum likelihood parameter estimation method to determine $p(\mathbf{x}\,|\,\boldsymbol{\theta}_1)$ and $p(\mathbf{x}\,|\,\boldsymbol{\theta}_2)$ (10 points)

6.2 If $P(\omega_1) = 0.5$ and $P(\omega_2) = 0.5$, use the minimum error criterion to determine :he decision function. (10 points)

5. Given a 3-state hidden Markov models defined by a transitional probability matrix and a symbol emission probability matrix as shown below:

$$A = \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0.1 & 0.2 & 0.2 & 0.2 & 0.3 \\ 0.1 & 0.2 & 0.1 & 0.4 & 0.2 \\ 0.4 & 0.2 & 0.1 & 0.2 & 0.1 \end{bmatrix}$$

Let the set of possible emitting symbols V_k be {a, b, c, d, e}

Ignore the entry and exit states and assume that state 1 is the first state and state 3 is the last state. Answer the following questions:

a) Sketch the structure of the HMM and label each link with the appropriate probability. (5 points)

b) Using the Forward algorithm to determine $P(V^T)$ where V^T = {c, b, a} (15 points)

c) From part (b), determine the most likely state sequence. (5 points)

__________ End of Exam ___________