มหาวิทยาลัยสงขลานครินทร์

คณะวิศวกรรมศาสตร์

สอบกลางภาค ประจำภาคการศึกษา 2 ปีการศึกษา 2550

วันที่ 6 มกราคม 2551 เวลา 09.00 – 12.00. ห้องสอบ R 300

วิชา 215-612: Finite Element Method, 220-504: Introduction to Finite Element Method

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ชื่อ-สกุล.....

รหัส.....

คำชี้แจง

1.ข้อสอบทั้งหมดมี 4 ข้อ คะแนนรวม 100 คะแนน ดังแสดงในตารางข้างล่าง

2.ข้อสอบมีทั้งหมด 4 หน้า (รวมปก) ผู้สอบต้องตรวจสอบว่ามีครบทุกหน้าหรือไม่ (ก่อนลงมือทำ)

3.ให้ทำหมดทุกข้อลงในสมุดคำตอบ

4.อนุญาตให้ใช้เครื่องคิดเลขได้ทุกชนิด

5.ห้ามหยิบ หรือยืมสิ่งของใดๆ ของผู้อื่นในห้องสอบ

6. ***Open Books***

7. ***GOOD LUCK***

ตารางคะแนน

ข้อที่	คะแนนเต็ม	ได้
1	25	
2	25	
3	25	
4	25	
รวม	100	

Problem 1 (25 Points)

Consider a boundary value problem:

$$\frac{d^2\phi}{dx^2} - \phi = 0 \quad , \quad 0 < x < 1$$
$$\phi(0) = 0$$
$$\frac{d\phi(1)}{dx} = 10$$

Let $\phi \approx \tilde{\phi} = \sum_{m=1}^{M} a_m N_m$ where a set N_m is selected such that the condition at x = 0 is automatically satisfied.

(a) Write the weighted residual statement of this problem. (5 points)

(b) If $N_m = x^m (m = 1,2,...,M)$, use the appropriate weighting function W_l (l=1,2,...,M) to obtain $\tilde{\phi}(x)$ when

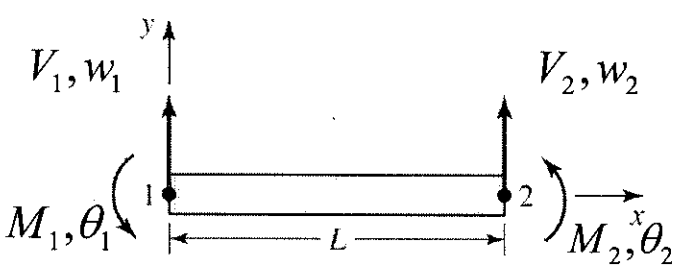
- $W_l = \delta(x - x_l)$; point collocation
- $W_l = N_l$; weak form of Galerkin's equation

Use M = 2. (20 points)

(c) Solve this differential equation analytically. (5 points)

Problem 2 (25 Points)

Consider a thin beam element with the governing equation and boundary equations being as shown below



$$EI\frac{d^4w}{dx^4} = 0 \ , 0 < x < \mathrm{L}$$
$$EI\frac{d^3w}{dx^3} = V_1 \ , x = 0$$
$$-EI\frac{d^2w}{dx^2} = M_1 \ , x = 0$$
$$-EI\frac{d^3w}{dx^3} = V_2 \ , x = \mathrm{L}$$
$$EI\frac{d^2w}{dx^2} = M_2 \ , x = \mathrm{L}$$

(a) Derive the total potential energy of this beam. Then, from the variational principle, formulate the stiffness matrix **K** and the force vector **F** corresponding

to $\{w_1 \quad \theta_1 \quad w_2 \quad \theta_2\}$ where $\theta = \frac{dw}{dx}$. (Let the deflection of this beam be given by a cub c approximation as $w(x) = a_1 + a_2 x + a_3 x^2 + a_4 x^3$)

(b) Show that we can obtain the matrices **K** and **F** by using the weak form of Galerkin's equation which are identical to the ones in (a).

(c) Show that the operator $\frac{d^4(.)}{dx^4}$ is self-adjointed.

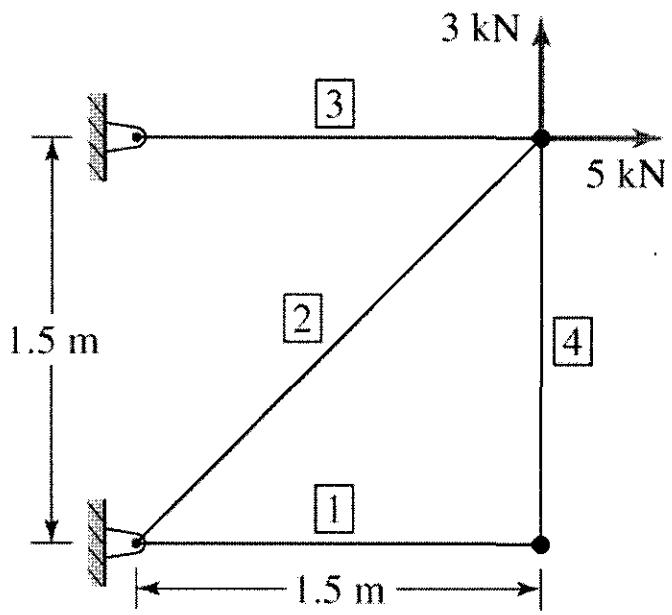
Problem 3 (25 Points)

The plane truss shown below is composed of members having a square 15 mm x 5 mm cross section and modulus of elasticity E = 69 GPa.

(a) Assemble the global stiffness matrix.

(b) Compute the nodal displacements in the global coordinate system for the loads shown.

(c) Compute the axial stress in each element, support reactions and also check the equilibrium of the system.



Problem 4 (25 Points)

A cylindrical rod that is one of several in a small heat exchange device is shown in the following figure. The left end of the pin is subjected to a constant temperature of 180 F^0 and the right end id in contact with a chilled water bath maintained at constant temperature of 40 F^0. The exterior surface of pin is in contact with moving air at 72 F^0.

The physical data are given as:

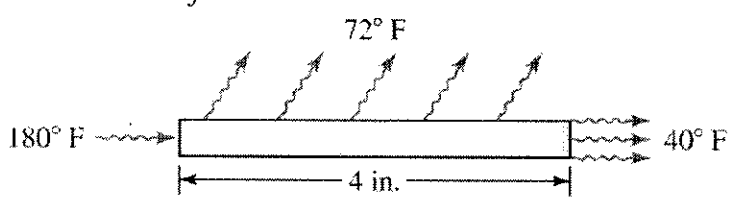
$$k = 120\ \frac{Btu}{hr - ft - F^0} : Thermal\ Conductivity$$

$$D = 0.5\ in. : Diameter\ of\ Pin$$

$$L = 4\ in. : Length\ of\ Pin$$

$$\beta_{air} = 50\ \frac{Btu}{hr - ft^2 - F^0} : Heat\ Transfer\ Coefficient\ of\ Air$$

$$\beta_{water} = 100\ \frac{Btu}{hr - ft^2 - F^0} : Heat\ Transfer\ Coefficient\ of\ Water$$



(a)

Use four equal-length linear elements to obtain a finite element solution for the temperature distribution across the length of the pin.