มหาวิทยาลัยสงขลานครินทร์

คณะวิศวกรรมศาสตร์

สอบปลายภาค ประจำภาคการศึกษา 2 ปีการศึกษา 2550

วันที่ 18 กุมภาพันธ์ 2551 ห้องสอบ R 300 เวลา 13.30 – 16.30.

วิชา 215-621: Finite Element Method, CE 220-504: Introduction to Finite Element Method

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ชื่อ-สกุล...

รหัส...........................

คำชี้แจง

1.ข้อสอบทั้งหมดมี 6 ข้อ คะแนนรวม 80 คะแนน ดังแสดงในตารางข้างล่าง

2.ข้อสอบมีทั้งหมด 5 หน้า (รวมปก) ผู้สอบต้องตรวจสอบว่ามีครบทุกหน้าหรือไม่ (ก่อน ลงมือทำ)

3.ให้ทำหมดทุกข้อลงในสมุดคำตอบ

4.อนุญาตให้ใช้เครื่องคิดเลขได้ทุกชนิด

5.ห้ามหยิบ หรือยืมสิ่งของใดๆ ของผู้อื่นในห้องสอบ

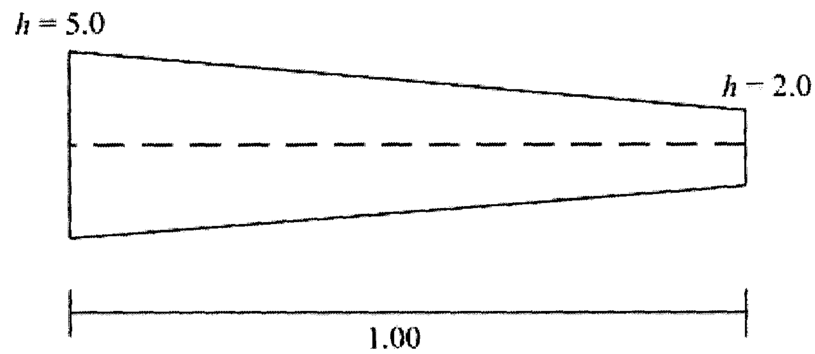
6. **Open Books**

ตารางคะแนน

ข้อที่	คะแนนเต็ม	ได้
1	10	
2	20	
3	20	
4	5	
5	10	
6	15	
รวม	80	

Problem 1 (10 Points)

To determine the element stiffness matrix of a tapered beam,



whose width is constant $b = 1$, but depth h varies linearly over the element as shown in the figure.

We have to integrate

$$\mathbf{K} = \int_0^1 B^2(x) EI(x) dx$$

where $B(x) = 6 - 12x$.

Use Gauss quadrature to integrate this problem by using the minimum n-point that can get the exact result.

Problem 2 (20 Points)

In the finite element formulation of near incompressible isotropic materials (as well as plasticity and viscoelasticity) it is convenient to use the so-called *Lame constants* λ and μ instead of E and v in the constitutive equations. Both λ and μ have the physical dimension of stress and are related to E and v by

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}, \quad \mu = G = \frac{E}{2(1+\nu)}.$$

Conversely

$$E = \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu}, \quad \nu = \frac{\lambda}{2(\lambda + \mu)}.$$

Substitute the second equation into the following equation

$$\text{plane stress:} \quad \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ 2e_{xy} \end{bmatrix},$$

$$\text{plane strain:} \quad \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & 0 \\ \frac{\nu}{1-\nu} & 1 & 0 \\ 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ 2e_{xy} \end{bmatrix}.$$

to express the two stress-strain matrices in terms of λ and μ. Then split the stress-strain matrix $\mathbf{E}$ of plane strain as

$$\mathbf{E}=\mathbf{E}_\mu+\mathbf{E}_\lambda$$

in which $\mathbf{E}_\mu$ and $\mathbf{E}_\lambda$ contain only μ and λ, respectively, with $\mathbf{E}_\mu$ diagonal and $E_{\lambda 33}=0$. This is the Lame or $\{\lambda\,,\,\mu\}$ splitting of the plane strain constitutive equations, which leads to the so-called **B**-bar formulation of near-incompressible finite elements Express $\mathbf{E}_\mu$ and $\mathbf{E}_\lambda$ also in terms of E and v.

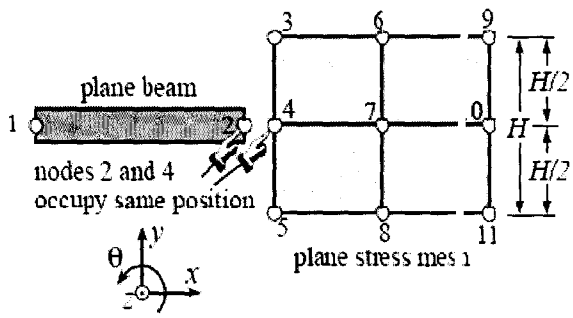
For the plane stress case perform a similar splitting in which where $\mathbf{E}_\lambda$ contains only $\bar{\lambda}=2\lambda\mu/(\lambda+2\mu)$ with $E_{\lambda 33}=0$, and $\mathbf{E}_\mu$ is a diagonal matrix function of μ and $\bar{\lambda}$. Express $\mathbf{E}_\mu$ and $\mathbf{E}_\lambda$ also in terms of E and v.

Problem 3 (20 Points)

A cantiveler beam-column is joined to a plane stress plate mesh as depicted in the figure. Both pieces move in the plane $\{x, y\}$. Plane stress elements have two degrees of freedom per node: two translations u_x and u_y along x and y, respectively, whereas a beam-column element has three: two translations u_x and u_y along x and y, and one rotation (positive CCW) θ_z about z. To connect the cantilever beam to the mesh, the following "gluing" conditions are applied:

(1) The horizontal (u_x) and vertical (u_y) displacements of the beam at their common node (2 of beam, 4 of plate) are the same.

(2) The beam end rotation θ_2 and the mean rotation of the plate edge 3–5 are the same. For infinitesimal displacements and rotations the latter is $\theta_{35}^{avg}=(u_{x5}-u_{x3})/H$.



Questions:

(a) Write down the three MFC conditions: two from (1) and one from (2), and state whether they are linear and homogeneous.

(b) Where does the above expression of θ_{35}^{avg} come from? (Geometric interpretation is ok.) Can it be made more accuracy by including u_{x4}? (To answer this question, observe that the displacements along 3-4 and 4-5 vary linearly. Thus the angle of rotation about z is constant for each of them and (for infinitesimal displacements) may be set equal to the tangent.)

(c) Write down the master-slave transformation matrix if $\{u_{x2}, u_{y2}, \theta_2\}$ are picked as slaves. It is sufficient to write down the transformation for the DOFs of nodes 2,3,4 and 5, which give a **T** of order 9×6, since the transformations for the other freedoms are trivial.

(d) If the penalty method is used, write down the stiffness equations of the three penalty elements assuming the same weight w is used. Their stiffness matrices are of order 2 × 2, 2 × 2 and 3 × 3, respectively. (Do not proceed further)

(e) If Lagrange multiplier adjunction is used, how many Lagrange multipliers will you need to append? (Do not proceed further)

Problems 4 (5 Points)

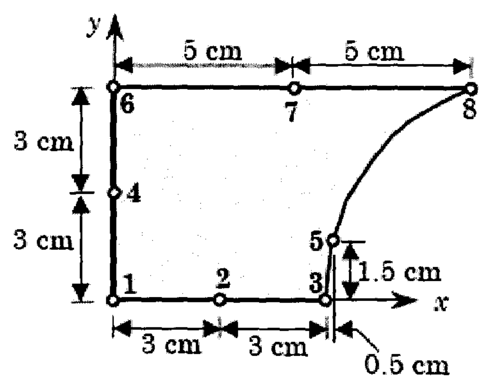
The free-free stiffness equations of a superelement are

$$\begin{bmatrix} 88 & -44 & -44 & 0 \\ -44 & 132 & -44 & -44 \\ -44 & -44 & 176 & -44 \\ 0 & -44 & -44 & 220 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 15 \\ 20 \end{bmatrix}$$

Eliminate u_2 and u_3 from the above equation by static condensation (using the explicit inverse formulas), and show (but do not solve) the condense equation system.

Problem 5 (10 Points)

Find the transformation equation, the Jacobian matrix for the eight-node element shown in the following figure.



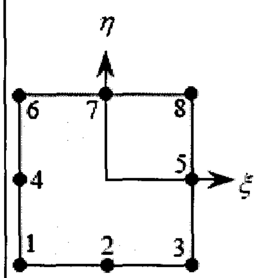
Hint:

The transformation equations are (contiued)

$$x = \sum_{i=1}^{8} x_i \psi_i(\xi, \eta)$$

$$y = \sum_{i=1}^{8} y_i \psi_i(\xi, \eta)$$

The corresponding shape functions of the eight-node rectangular element.



$$\psi_1 = -\tfrac{1}{4}(1-\xi)(1-\eta)(1+\xi+\eta)$$
$$\psi_2 = \tfrac{1}{2}(1-\xi^2)(1-\eta)$$
$$\psi_3 = -\tfrac{1}{4}(1+\xi)(1-\eta)(1-\xi+\eta)$$
$$\psi_4 = \tfrac{1}{2}(1-\xi)(1-\eta^2), \quad \psi_5 = \tfrac{1}{2}(1+\xi)(1-\eta^2)$$
$$\psi_6 = -\tfrac{1}{4}(1-\xi)(1+\eta)(1+\xi-\eta)$$
$$\psi_7 = \tfrac{1}{2}(1-\xi^2)(1+\eta)$$
$$\psi_8 = -\tfrac{1}{4}(1+\xi)(1+\eta)(1-\xi-\eta)$$

Problem 6 (15 Points)

Construct the shape functions of the 6-node triangular element (x,y co-ordinate). Show intermediate steps of calculation.