มหาวิทยาลัยสงขลานครินทร์

คณะวิศวกรรมศาสตร์

สอบปลายภาค ประจำภาคการศึกษา 2 ปีการศึกษา 2551

วันที่ 20 กุมภาพันธ์ 2552 ห้องสอบ A 401 เวลา 13.30 – 16.30.

วิชา CE 220-504: Introduction to Finite Element Method

ชื่อ-สกุล...

รหัส...........................

คำชี้แจง

1.ข้อสอบทั้งหมดมี 6 ข้อ คะแนนรวม 70 คะแนน ดังแสดงในตารางข้างล่าง

2.ข้อสอบมีทั้งหมด 5 หน้า (รวมปก) ผู้สอบต้องตรวจสอบว่ามีครบทุกหน้าหรือไม่ (ก่อนลงมือทำ)

3.ให้ทำหมดทุกข้อลงในสมุดคำตอบ

4.อนุญาตให้ใช้เครื่องคิดเลขได้ทุกชนิด

5.ห้ามหยิบ หรือยืมสิ่งของใด ๆ ของผู้อื่นในห้องสอบ

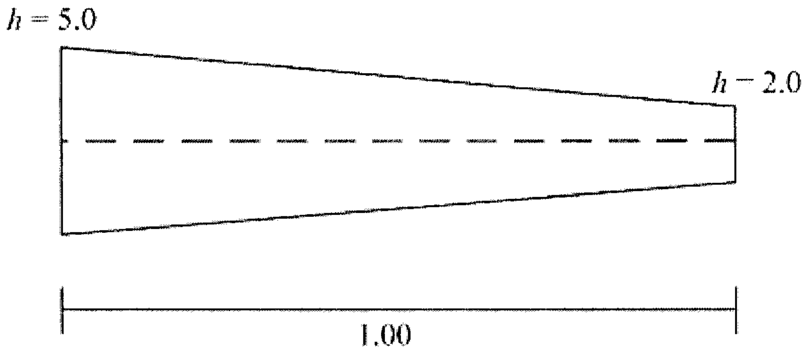
6. **Open Books**

ตารางคะแนน

ข้อที่	คะแนนเต็ม	ได้
1	15	
2	10	
3	5	
4	10	
5	15	
6	15	
รวม	70	

Problem 1 (15 Points)

To determine the element stiffness matrix of a tapered beam,



whose width is constant b = 1, but depth h varies linearly over the element as shown in the figure.

We have to integrate

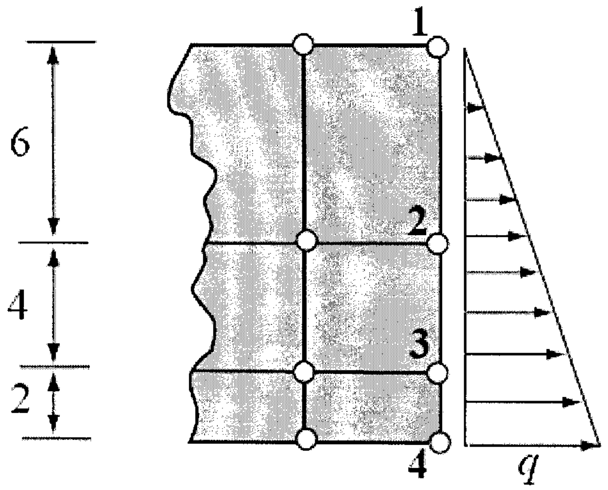
$$\mathbf{K} = \int_0^1 B^2(x) EI(x) dx$$

where $B(x) = 6 - 12x$.

Use Gauss quadrature to integrate this problem by using the minimum n-point that can get the exact result.

Problems 2 (10 Points)

Compute the "lumped" nodal forces f_1, f_2, f_3 and f_4 equivalent to the linearly-varying distributed surface load q for the finite element layout defined in the following figure. Use both NbN and EbE lumping. For example, $f_1 = 3q/8$ for NbN. Check that $f_1 + f_2 + f_3 + f_4 = 6q$ for both schemes (why?). Note that q is given as a force per unit of vertical length.



Problems 3 (5 Points)

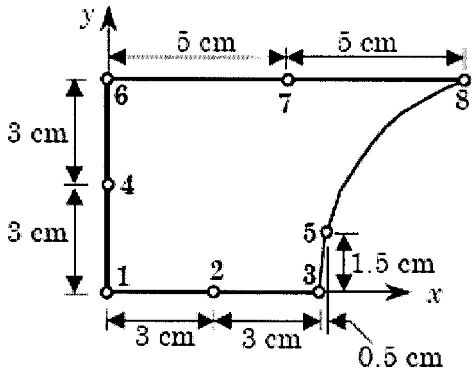
The free-free stiffness equations of a superelement are

$$\begin{bmatrix} 88 & -44 & -44 & 0 \\ -44 & 132 & -44 & -44 \\ -44 & -44 & 176 & -44 \\ 0 & -44 & -44 & 220 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 15 \\ 20 \end{bmatrix}$$

Eliminate u_2 and u_3 from the above equation by static condensation (using the explicit inverse formulas), and show (but do not solve) the condense equation system.

Problem 4 (10 Points)

Find the transformation equation, the Jacobian matrix for the eight-node element shown in the following figure.



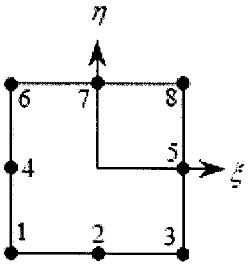
Hint:

The transformation equations are (contiued)

$$x = \sum_{i=1}^{8} x_i \psi_i(\xi, \eta)$$

$$y = \sum_{i=1}^{8} y_i \psi_i(\xi, \eta)$$

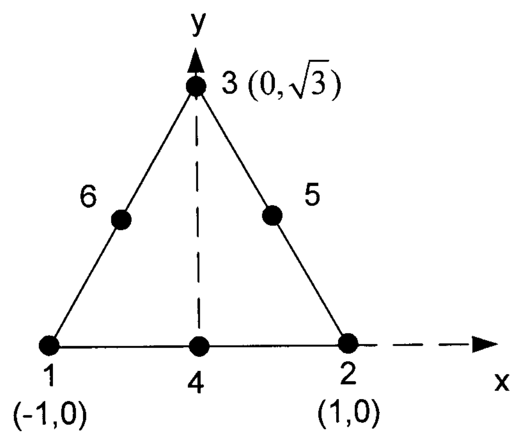
The corresponding shape functions of the eight-node rectangular element.



$$\psi_1 = -\tfrac{1}{4}(1-\xi)(1-\eta)(1+\xi+\eta)$$
$$\psi_2 = \tfrac{1}{2}(1-\xi^2)(1-\eta)$$
$$\psi_3 = -\tfrac{1}{4}(1+\xi)(1-\eta)(1-\xi+\eta)$$
$$\psi_4 = \tfrac{1}{2}(1-\xi)(1-\eta^2), \quad \psi_5 = \tfrac{1}{2}(1+\xi)(1-\eta^2)$$
$$\psi_6 = -\tfrac{1}{4}(1-\xi)(1+\eta)(1+\xi-\eta)$$
$$\psi_7 = \tfrac{1}{2}(1-\xi^2)(1+\eta)$$
$$\psi_8 = -\tfrac{1}{4}(1+\xi)(1+\eta)(1-\xi-\eta)$$

Problem 5 (15 Points)

Construct the shape functions of the 6-node triangular element (x,y co-ordinate). Show intermediate steps of calculation.



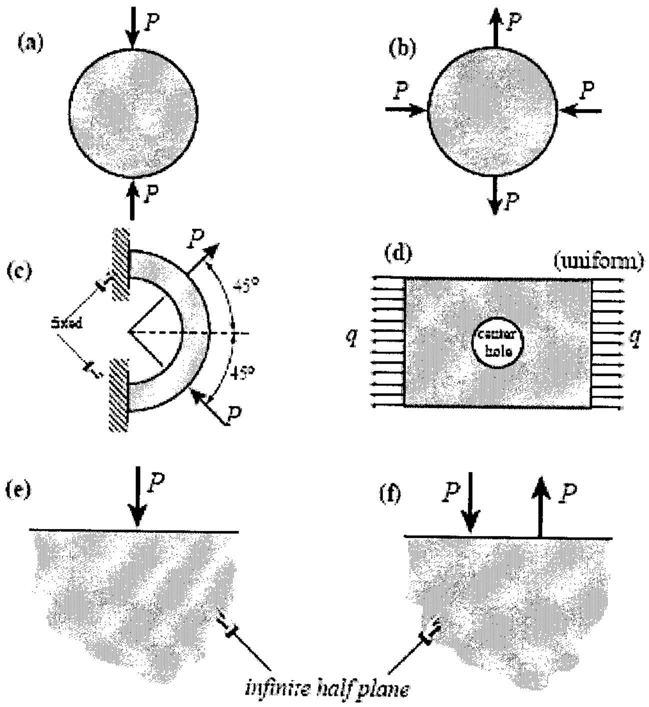
Problem 6 (15 Points)

Identify the symmetry and antisymmetry lines in the two-dimensional problems illustrated in the following figure. They are:

(a) a circular disk under two diametrically opposite point forces (the famous "Brazilian test" for concrete)
(b) the same disk under two diametrically opposite force pairs
(c) a clamped semi annulus under a force pair oriented as shown
(d) a stretched rectangular plate with a central circular hole.
(e) and (f) are half-planes under concentrated loads.

Having identified those symmetry/antisymmetry lines, state whether it is possible to cut the complete structure to one half or one quarter before laying out a finite element mesh. Then draw a coarse FE mesh indicating, with rollers or fixed supports, which kind of displacement BCs you would specify on the symmetry or antisymmetry lines.

Note: Do all sketches on your paper, not on the printed figures.


(a)
P
P
(b)
P
P
P
P
(c)
P
45°
fixed
45°
P
(d)
(uniform)
q
center hole
q
(e)
P
(f)
P
P
infinite half plane