PRINCE OF SONGKLA UNIVERSITY
FACULTY OF ENGINEERING

Final Exam : Semester I — Academic Year : 2009

Date : September 29, 2009 — Time : 13:30-16:30

Subject : Advanced Mathematics for Chemical Engineering (230-600)

Total pages : 13 (inc. front page) — Room : S104

NAME .... STUDENT ID ....

ไม่อนุญาตให้นำเอกสารและเครื่องคำนวณทุกชนิดเข้าห้องสอบ

ทุจริตในการสอบโทษขั้นต่ำคือปรับตกในรายวิชาที่ทุจริต

และพักการศึกษา 1 ภาคการศึกษา

Question #	Total Score	Score
1	20	
2	15	
3	20	
4	25	
5	20	
6	20	
Total	**120**	

ดร. พรศิริ แก้วประดิษฐ์

ผู้ออกข้อสอบ

1. (20 points) The motion of a mass on a spring moving in a medium, in which damping is negligible, can be described as following:

$$\frac{d^2y}{dt^2} + y = 4\delta(t - 2\pi)$$

At $t = 2\pi$, the mass is given a sharp blow. (solve by using Laplace Transform – invert using **partial fraction**).

1.1. (10 points), If the mass is at rest in the equilibrium position, $y(0) = 0$ and $\left.\frac{dy}{dt}\right|_{t=0} = 0$, find the function of y(t) at $0 \leq t < 2\pi$ and $t \geq 2\pi$

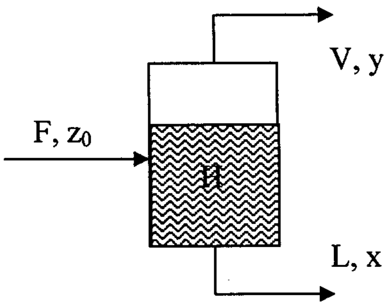
1.2. (10 points), If the mass is released from rest 1 unit below the equilibrium position, $y(0) = 1$ and $\left.\frac{dy}{dt}\right|_{t=0} = 0$, <u>find the function of y(t)</u> at $0 \leq t < 2\pi$ and $t \geq 2\pi$

2. (15 points) The rate equation for component B involved in the following first – order reactions, $A \overset{k_1}{\rightarrow} B \overset{k_2}{\rightarrow} C$, can be written as,

$$\frac{dc_B}{dt} + k_2 c_B = k_1 c_{A0} e^{-k_1 t}$$

Determine the concentration of B as a function of time, if the condition $c_B = 0$ at $t = 0$ (by Laplace transform – invert using **convolution**).

3. (20 points) A feed solution containing z_0 mass fraction of a particular volatile component is fed to a single effect evaporator at a rate of F kg/hr. It may assume that vapor rate (V) and liquid rate (L) are constant and the liquid holdup in the evaporator is H kg. The dynamic mass balance can be written as,



$$H\frac{dx(t)}{dt} = Fz_0(t) - Vy(t) - Lx(t)$$

and $y(t) = mx(t)$ where, y = the equilibrium mass fraction in vapor, x = the mass fraction of the volatile component in liquid. **It is noted that**:

$\alpha = \frac{mV+L}{H}, \beta = \frac{F}{H}$

3.1. (10 points), <u>find dynamic equation in deviation form,</u> $\frac{d\hat{x}}{dt}$,

where $\hat{x} = x - \bar{x}$ and $\bar{x}$ = liquid volatile mass fraction at steady-state condition

3.2. (10 points), find response x(t), when inlet mass fraction takes impulse disturbance, $c_0\delta(t)$, (solve by using **Laplace Transform**)

4. (25 points) Heat diffusion equation, T(x,t) in a semi-infinite slab can be described in new variable $\theta(x,t)$ as,

$$\alpha\frac{\partial^2\theta}{\partial x^2}=\frac{\partial\theta}{\partial t}$$

where, $\theta(x,t) = T(x,t) - T_0$ and $T(x,0) = T_0$, $T(0,t) = T_s$, $T(\infty,t) = T_0$

4.1. (2 points), <u>find the initial conditions</u> in term of $\theta(x,t)$

4.2. (3 points), <u>find the boundary conditions</u> in term of $\theta(x,t)$

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4.2. (3 points), <u>find the boundary conditions</u> in term of $\theta(x,t)$

4.3. (20 points), <u>determine the particular solution T(x,t)</u> by using **Laplace Transform**

5. (20 points) The one – dimension linear wave equation for $u(x,t)$ can be written as,

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

where, $x \in [0,1]$, subject to the initial conditions,

$$u(x, t = 0) = \sin(\pi x)$$

and

$$\left.\frac{\partial u}{\partial t}\right|_{t=0} = 0$$

and the boundary conditions,

$$u(x = 0, t) = 0 \quad , \qquad u(x = 1, t) = 0$$

Find the solution of the problem by using **a separation of variables**

6. (20 points) Find solution of the following PDE by using **combination variables method,**

$$\frac{1}{k}u_t = u_{xx}$$

If the combination variable is dimensionless, $\eta = \frac{x^2}{kt}$

Table of Laplace Transforms

F(s)	f(t)
$\frac{1}{s}$	$u(t) = 1$
1	$\delta(t)$
$\frac{1}{s^2}$	t
$\frac{1}{s+a}$	e^{-at}
$\frac{1}{(s+a)^2}$	te^{-at}
$\frac{1}{s^2+a^2}$	$\frac{1}{a}\sin(at)$
$\frac{s}{s^2+a^2}$	$\cos(at)$
$\frac{1}{s^2-a^2}$	$\frac{1}{a}\sinh(at)$
$\frac{s}{s^2-a^2}$	$\cosh(at)$
$\frac{1}{s}e^{-\alpha\sqrt{s}}, \quad (\alpha \geq 0)$	$\operatorname{erfc}\left(\frac{\alpha}{2\sqrt{t}}\right)$
$e^{-as}F(s)$	$f(t-a)u(t-a)$
$F(s-a)$	$e^{-at}f(t)$

Fourier cosine series:

$$f(x) = \sum_{n=0}^{\infty} a_n \cos\left(\frac{n\pi}{L}x\right) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{L}x\right)$$

$$a_0 = \frac{1}{L}\int_0^L f(x)dx \quad \text{and} \quad a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi}{L}x\right)dx \quad , \quad n = 1,2\ldots$$

Fourier sine series:

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{L}x\right)$$

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx \quad , \quad n = 1,2\ldots$$