มหาวิทยาลัยสงขลานครินทร์

คณะวิศวกรรมศาสตร์

สอบกลางภาค ประจำภาคการศึกษา 2 ปีการศึกษา 2552

วันที่ 23 ธันวาคม 2552 เวลา 9.00 – 16.30.

วิชา CE 220-504: Introduction to Finite Element Method ห้องสอบ CE 106

ชื่อ-สกุล...

รหัส...............................

คำชี้แจง

1.ข้อสอบทั้งหมดมี 6 ข้อ คะแนนรวม 300 คะแนน ดังแสดงในตารางข้างล่าง

2.ข้อสอบมีทั้งหมด 5 หน้า (รวมปก) ผู้สอบต้องตรวจสอบว่ามีครบทุกหน้าหรือไม่ (ก่อนลงมือทำ)

3.ให้ทำหมดทุกข้อลงในสมุดคำตอบ

4.อนุญาตให้ใช้เครื่องคิดเลขได้ทุกชนิด

5.ห้ามหยิบ หรือยืมสิ่งของใดๆ ของผู้อื่นในห้องสอบ

6. ***Open Books***

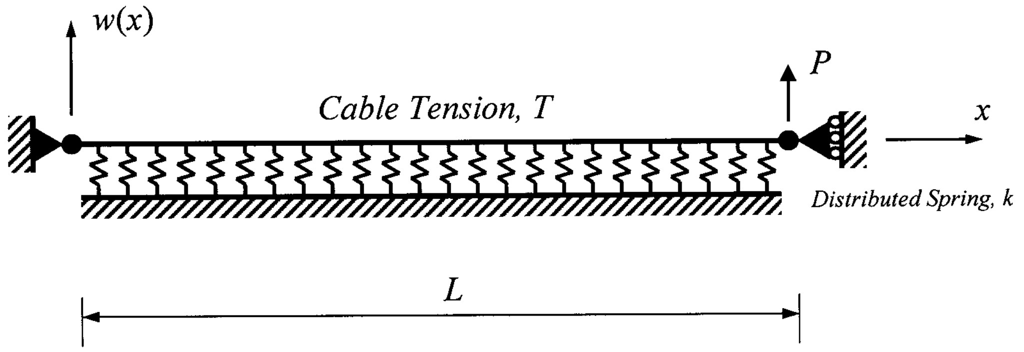
7. ***GOOD LUCK***

ตารางคะแนน

ข้อที่	คะแนนเต็ม	ได้
1	50	
2	50	
3	50	
4	50	
5	50	
6	50	
รวม	300	

Problem 1 (50 Points)

A prestressed cable with tension, T, has a support with a distributed stiffness of k. The load is a vertical force to a load P at the end free to translate vertically. Axial deformation of the cable is neglected.



(a) Show that the governing differential equation of equilibrium (strong form) for the vertical displacement, $w(x)$ of the cable in term of k, T, and P is:

$$T\frac{d^2w(x)}{dx^2} - kw(x) = 0 \quad for \quad 0 < x < L$$

Hint: for small displacements, we have:

$\cos\alpha \approx 1$ and $\sin\alpha \approx \alpha$

(b) Show that the boundary conditions needed to solve the above differential equation are:

Essential Boundary Condition: $w(0) = 0$

Natural Boundary Condition: $Tw'(L) = P$

(c) Starting with the strong form of equilibrium, determine the weak form and identify the primary and secondary variables.

(d) Using the weak form, construct a finite element equation for the problem.

(e) Develop a two-node finite element, and use three elements (each L/3 long) to solve the problem.

Assume that $k = T/L^2$ and $P = T = L = 1$

Problem 2 (50 Points)

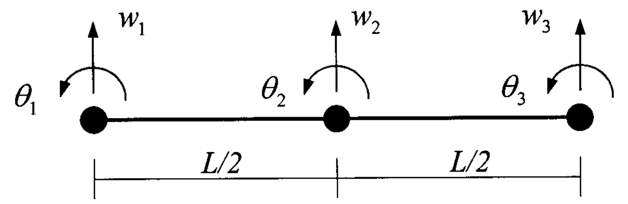
For the Euler-Bernoulli beam with constant IE shown in Figure below,

Derive

(a) The interpolation functions

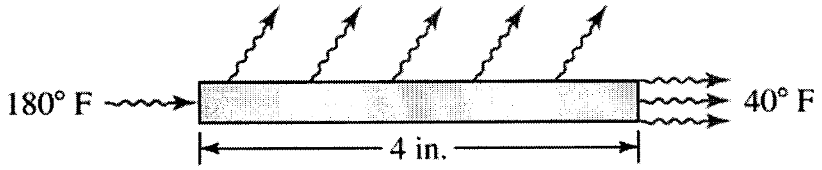
(b) The full element stiffness matrix (6x6)

(c) The reduced element stiffness matrix (4x4) obtained by eliminating the internal degrees of freedom.



Problem 3 (50 Points)

A cylindrical rod that is one of several in a small heat exchange device is shown in Figure below. The left end of the pin is subjected to a constant temperature of 180 F^0 and the right end is in contact with a chilled water bath maintained at constant temperature of 40 F^0. The exterior surface of pin is in contact with moving air at 72 F^0.



The physical data are given as:

$$k = 120 \frac{Btu}{hr - ft - F^0} : Thermal\ Conductivity$$

$$D = 0.5\ in.: Diameter\ of\ Pin$$

$$L = 4\ in.: Length\ of\ Pin$$

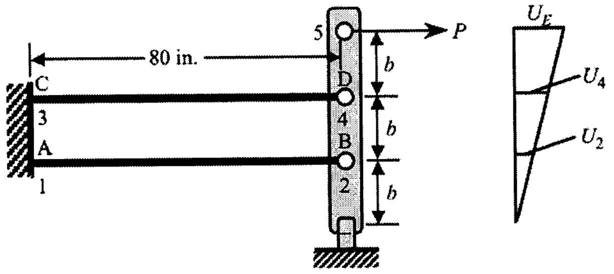
$$\beta_{air} = 50 \frac{Btu}{hr - ft^2 - F^0} : Heat\ Transfer\ Coefficient\ \ of\ Air$$

$$\beta_{water} = 100 \frac{Btu}{hr - ft^2 - F^0} : Heat\ Transfer\ Coefficient\ \ of\ Water$$

Problem 4 (50 Points)

For the structure shown in Figure below, determine the forces and elongation in rods AB and CD. Each rod has a cross-sectional area A of 0.03 in^2 and modulus of elasticity $E = 30\times10^6\,psi$.

Please use ***Penalty Method*** to cope with the kinematics constraint.



Problem 5 (50 Points)

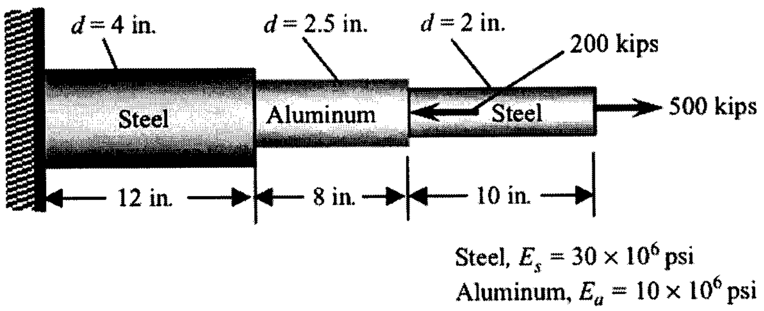
The governing differential equation is of the form:

$$-\frac{d}{dx}\left[EA\frac{du}{dx}\right] = 0;\ 0 < x < L$$

For the minimum number of linear elements, give

(a) the boundary conditions on the nodal variables (primary as well as secondary)

(b) the final condensed finite element equations for the unknowns



Problem 6 (50 Points)

Consider the differential equation

$$-\frac{d^2u}{dx^2} = \cos \pi x : 0 < x < 1$$

and subjected to the following boundary conditions:

$$u(0) = 0 \text{ and } u(1) = 0$$

Use the uniform mesh of three linear elements to solve the problem