PRINCE OF SONGKLA UNIVERSITY
FACULTY OF ENGINEERING

Midterm Examination: Semester II	Academic Year: 2009
Date: 24 December 2009	Time: 13:30-16:30
Subject: 241-650 Principles of Pattern Recognition	Room: R300

Instructions:

This exam has 6 problems, 10 pages and 65 points. You may use the back of the pages for scratch work. This exam is closed books and notes. You are allowed to use a calculator. You may consult one A4 sheet of notes (two sides).

Name .... ID ....

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1. A pattern is represented by a pair of features, $(x_1,x_2)^T$. Suppose there are 2 classes of patterns, each having a mean vector and a covariance matrix shown in the table below:

Class	Mean	Covariance
1	$(1, 2)^T$	$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$
2	$(5, 3)^T$	$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

Use the technique of minimum distance classifier to determine the equation of the decision boundary, assuming equal prior probability (10 points)

2. Two random variables x and y are called statistically independent if $p(\mathbf{x},\mathbf{y} \mid \omega) = p(\mathbf{x} \mid \omega) p(\mathbf{y} \mid \omega)$. Prove that if $\mathbf{x}_i - \boldsymbol{\mu}_i$ and $\mathbf{x}_j - \boldsymbol{\mu}_j$ are statistically independent (for $i \neq j$) then $\sigma_{ij} = \mathcal{E}\left[(\mathbf{x}_i - \boldsymbol{\mu}_i)(\mathbf{x}_j - \boldsymbol{\mu}_j)\right] = 0$
($\boldsymbol{\mu}_i, \boldsymbol{\mu}_j$ are constants) (10 points)

3. Let x has a uniform density $p(x\mid\theta)=U(0,\theta)=\begin{cases}\frac{1}{\theta} & 0\le x\le\theta \\ 0 & otherwise\end{cases}$

Suppose that n samples $\boldsymbol{D} = \{x_1, x_2, \dots, x_n\}$ are drawn independently according to $p(x\mid\theta)$. Show that the maximum likelihood estimate for θ is *max*[***D***]—that is, the value of the maximum element in ***D***. (10 points)

4. Suppose we have 3 classes of patterns whose features are in 2-D feature space. Each has the following underlying distributions:

$$p(\mathbf{x} \mid \omega_1) \cong N(\mathbf{0}, 2\mathbf{I})$$

$$p(\mathbf{x} \mid \omega_2) \cong N\left(\begin{bmatrix} 2 \\ 2 \end{bmatrix}, \mathbf{I}\right)$$

$$p(\mathbf{x} \mid \omega_3) \cong N\left(\begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, \mathbf{I}\right)$$

with $P(\omega_1) = 1/2$ and $P(\omega_2) = P(\omega_3) = 1/4$.

By explicit calculation of posterior probabilities, classify the point $x = \begin{bmatrix} 0.5 \\ 2 \end{bmatrix}$ for minimum probability of error. (15 points)

5. Class 1 and Class 2 has the feature vectors as shown in the following table:

Class 1		Class 2	
x1	x2	x1	x2
-3	0	6	0
-2	1	5	0
-2	0	5	2
-1	-1	4	-2

The features of each class are assumed to have a Gaussian distribution function (normally distributed). Use the maximum likelihood parameter estimation method to determine $p(\mathbf{x} \mid \boldsymbol{\theta}_1)$ and $p(\mathbf{x} \mid \boldsymbol{\theta}_2)$ (10 points)

6. Find the eigenvalues and eigenvectors of matrix **A**, where $\mathbf{A} = \begin{bmatrix} 1 & 3 \\ 1 & -1 \end{bmatrix}$ (10 points)