

PRINCE OF SONGKLA UNIVERSITY  
FACULTY OF ENGINEERING

Final Examination: Semester II  
Date: Feb 22, 2010  
Subject: 210-251 Electromagnetic Field Theory  
210-351 Electromagnetic Field Theory

Academic Year: 2009  
Time: 9.00-12.00  
Room: หัวหุ่นยนต์  
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**Instructions:**

- a. Allow a student to open his/her own note (**one A4-size paper**) during the exam
- b. Allow the student to use his/her own calculator and dictionary

**Do all four problems**

1. Faraday's law

1.1 The armature of a 60 Hz ac generator rotates in a 0.15 T magnetic field. If the area of the coil is  $2 \times 10^{-2} \text{ m}^2$ , how many loops must the coil contain if the peak output is to be  $\epsilon_{\text{max}} = 170 \text{ V}$ ?

1.2 The rectangular loop shown in Figure 1 has a constant width  $l$ , but its length  $x$  increases with time as a conducting bar slides at a uniform velocity  $v$  in a static magnetic field  $\mathbf{B}$ . Find the motional emf.

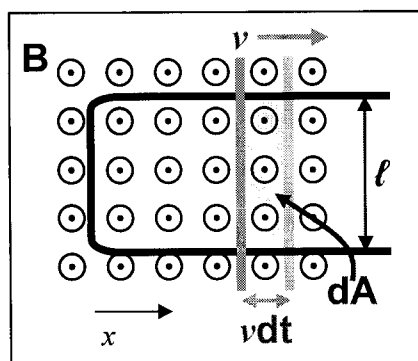


Figure 1 Sliding bar

(25 points)

2. The magnetic field intensity is given in the square region  $x=0, 0.5 < y < 1, 1 < z < 1.5$  by

$$\mathbf{H} = z^2 \mathbf{a}_x + x^3 \mathbf{a}_y + y^4 \mathbf{a}_z \text{ A/m.}$$

- a) Evaluate  $\oint \mathbf{H} \cdot d\mathbf{L}$  about the perimeter of the square region
- b) Find  $\nabla \times \mathbf{H}$ .
- c) Calculate  $(\nabla \times \mathbf{H})_x$  at the center of the region.
- d) Does  $(\nabla \times \mathbf{H})_x = (\oint \mathbf{H} \cdot d\mathbf{L}) / \text{Area enclosed}$ ?

(25 points)

3. In free space, where  $\epsilon = \epsilon_0, \mu = \mu_0, \sigma = 0, \rho_v = 0$  and  $\mathbf{J} = 0$ , assume a Cartesian coordinate system in which  $\mathbf{E}$  and  $\mathbf{H}$  are both functions only of  $z$  and  $t$ .

a) If  $\mathbf{E} = E_y \mathbf{a}_y$  and  $\mathbf{H} = H_x \mathbf{a}_x$ , begin with Maxwell's equations and determine the second-order partial differential equation that  $E_y$  must satisfy.

b) Let  $E_y = 5(300t + bz)^2$  be a solution of the equation in a) for a particular value of  $b$ . Find the value of  $b$ .

(25 points)

4. Poynting vector

a) Find the Poynting vector,  $P(\mathbf{r}, t)$  if  $E_s = 400e^{-j2x} \mathbf{a}_y$  V/m in free space

b) Find  $P$  at  $t=0$  for  $\mathbf{r}=(a, 5, 10)$ , where  $a = 0, 1, 2,$  and  $3$ .

c) Find  $P$  at the origin for  $t = 0, 0.2T, 0.4T$  and  $0.6T$ , where  $T$  is the oscillation period.

(25 points)

### List of vector identities

$$(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} \equiv (\mathbf{B} \times \mathbf{C}) \cdot \mathbf{A} \equiv (\mathbf{C} \times \mathbf{A}) \cdot \mathbf{B} \quad (\text{A.6})$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) \equiv (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C} \quad (\text{A.7})$$

$$\nabla \cdot (\mathbf{A} + \mathbf{B}) \equiv \nabla \cdot \mathbf{A} + \nabla \cdot \mathbf{B} \quad (\text{A.8})$$

$$\nabla(V + W) \equiv \nabla V + \nabla W \quad (\text{A.9})$$

$$\nabla \times (\mathbf{A} + \mathbf{B}) \equiv \nabla \times \mathbf{A} + \nabla \times \mathbf{B} \quad (\text{A.10})$$

$$\nabla \cdot (V\mathbf{A}) \equiv \mathbf{A} \cdot \nabla V + V\nabla \cdot \mathbf{A} \quad (\text{A.11})$$

$$\nabla(VW) \equiv V\nabla W + W\nabla V \quad (\text{A.12})$$

$$\nabla \times (V\mathbf{A}) \equiv \nabla V \times \mathbf{A} + V\nabla \times \mathbf{A} \quad (\text{A.13})$$

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) \equiv \mathbf{B} \cdot \nabla \times \mathbf{A} - \mathbf{A} \cdot \nabla \times \mathbf{B} \quad (\text{A.14})$$

$$\nabla(\mathbf{A} \cdot \mathbf{B}) \equiv (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A} + \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) \quad (\text{A.15})$$

$$\nabla \times (\mathbf{A} \times \mathbf{B}) \equiv \mathbf{A}\nabla \cdot \mathbf{B} - \mathbf{B}\nabla \cdot \mathbf{A} + (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B} \quad (\text{A.16})$$

$$\nabla \cdot \nabla V \equiv \nabla^2 V \quad (\text{A.17})$$

$$\nabla \cdot \nabla \times \mathbf{A} \equiv 0 \quad (\text{A.18})$$

$$\nabla \times \nabla V \equiv 0 \quad (\text{A.19})$$

$$\nabla \times \nabla \times \mathbf{A} \equiv \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} \quad (\text{A.20})$$