PRINCE OF SONGKLA UNIVERSITY

DEPARTMENT OF CHEMICAL ENGINEERING, FACULTY OF ENGINEERING

Examination paper: Midterm Exam 230-600 Semester: 1/2010

Date: Aug 1, 2010 Time: 9.00–12.00

Subject: Advanced Engineering Mathematics for Chemical Engineers Room: S 817

Instruction:

- **Calculator, books, notes and class materials are allowed.**
- **No talking or discussing during taking this exam.**

Items	Full scores	Your scores
1	20	
2	35	
3	45	
Total	100	

1. (20 points) Solve the following equations:

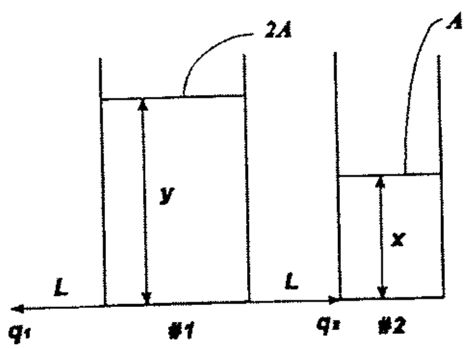
$$(a)\quad \left(\frac{3y^2 - x^2}{y^5}\right)\frac{dy}{dx} + \frac{x}{2y^4} = 0\ ; \qquad y(1) = 1$$

$$(b)\quad y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \frac{dy}{dx}\ ; \qquad y(0) = 1,\ \ y'(0) = 2$$

2. (35 points) Two vertical, cylindrical tanks, each 10 m high, are installed side-by-side in a tank farm, their bottoms at the same level. The tanks are connected at their bottoms by a horizontal pipe 2 meters long, with pipe inside diameter 0.03 m. The first tank (1) is full of oil and the second tank (2) is empty. Moreover, tank 1 has a cross-sectional area twice that of tank 2.
The first tank also has another outlet (to atmosphere) at the bottom, composed of a short horizontal pipe 2 m long, 0.03-m diameter. Both of the valves for the horizontal pipes are opened simultaneously.

Assume laminar flow in the horizontal pipes, and neglect kinetic, entrance-exit losses.

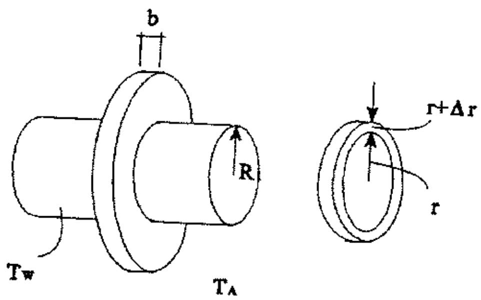
For laminar flow, the flow rates are given by $q_1 = \frac{ky}{L}$, $q_2 = \frac{k(y-x)}{L}$;k is a constant.



(a) Show that the volume balance ratio yields the equation: $\frac{1}{2}\frac{dx}{dy} = -\frac{q_2}{q_1+q_2}$.

(b) What is the maximum oil level (x_{max}) in tank 2?

3. (45 points) Thin, metallic circular fins of thickness b can be attached to cylindrical pipes as heat transfer promoters. The fins are exposed to an ambient temperature T_A, and the root of each fin contacts the pipe at position $r = R$, where the temperature is constant, T_w. The fin loses heat to ambient air through a transfer coefficient h. The metallic fin transmits heat by conduction in the radial direction.



(a) Show that the steady-state heat balance on an elementary annular element of fin yields the equation

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)-\left(\frac{2h}{bk}\right)(T-T_A)=0$$

(b) Define a dimensionless radial coordinate as $x = r\sqrt{\frac{2h}{bk}}$ and introduce $y = T - T_A$, and thus show the elementary equation

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-x^2y=0$$

describes the physical situation.

(c) Apply the method of Frobenius and find the roots of the indicial equation to show that $C_1 = C_2 = 0$.

(d) Complete the solution and show that the first few terms of the solution are

$$y=a_0\left[1+\left(\frac{x}{2}\right)^2+\left(\frac{x}{2}\right)^4\left(\frac{1}{2!}\right)^2+\ldots\right]$$

$$+b_0\left\{\ln(x)\left[1+\left(\frac{x}{2}\right)^2+\left(\frac{x}{2}\right)^4\frac{1}{(2!)^2}+\ldots\right]-\left[\left(\frac{x}{2}\right)^2+\frac{3}{2}\left(\frac{x}{2}\right)^4\frac{1}{(2!)^2}+\ldots\right]\right\}$$