



การสอบปลายภาค ประจำภาคการศึกษาที่ 1
วันที่ : 4 ตุลาคม 2553
วิชา : 241-306 Signal and Systems

ปีการศึกษา : 2553
เวลา : 9:00 – 12:00
ห้อง : S817

ทุจริตในการสอบ โทษขั้นต่ำคือ ปรับตกในรายวิชาที่ทุจริต และพักการเรียนหนึ่งภาคการศึกษา

คำสั่ง

- ข้อสอบมี 3 ตอน
ตอนที่ 1 มี 4 ข้อ 11 คะแนน
ตอนที่ 2 มี 3 ข้อ 14 คะแนน
ตอนที่ 3 มี 3 ข้อ 15 คะแนน
รวมทั้งหมด 19 หน้า (ไม่รวมปก) ให้นักศึกษาทำข้อสอบทุกตอน และทุกข้อ
- ห้ามนำเครื่องคิดเลขเข้าห้องสอบ
- ห้ามนำเอกสารใดๆ เข้าห้องสอบ
- แสดงวิธีทำและเขียนคำตอบให้ชัดเจน ถ้าอ่านไม่ออกถือว่าตอบผิด ไม่แสดงวิธีทำถือว่าตอบผิด
- ข้อสอบแต่ละข้อคะแนนไม่เท่ากัน

รหัสนักศึกษา : _____ ชื่อ : _____ ตอน : _____

คำถาม	1	2	3	4	5	6	รวม
ตอนที่ 1							
ตอนที่ 2							
ตอนที่ 3							

Student ID : _____ Name : _____ Section : _____

ตอนที่ 1 (11 คะแนน, 11 เปอร์เซนต์)

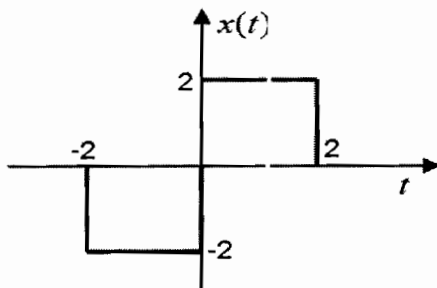
1. จงหา Fourier transform ของสัญญาณต่อไปนี้

1.1) $x(t) = 2\delta(t) - \delta(t - 2) + \delta(t + 2)$ (1 คะแนน)

ตอบ _____

1.2)

(1 คะแนน)



ตอบ _____

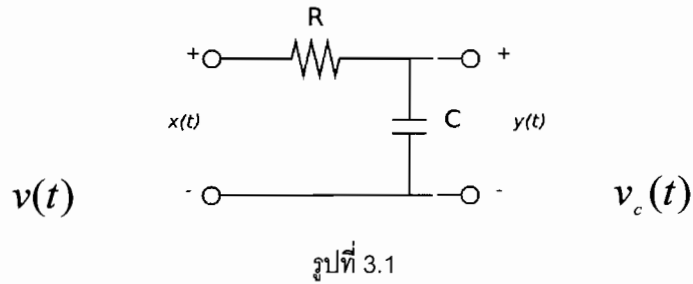
2. จงหา Fourier transform พร้อมทั้งวาดกราฟ $X(e^{j\omega})$ ของสัญญาณต่อไปนี้ (3 คะแนน)

$$x[n] = 1 + 2\cos(2n\pi + \frac{\pi}{2}) + 2\cos(4n\pi)$$

ตอบ _____

ตอนที่ 3 (15 คะแนน, 15 เปอร์เซ็นต์)

1. กำหนดวงจรกรองความถี่ต่ำ ซึ่งประกอบด้วย RC ดังแสดงในรูปที่ 3.1 โดย $v_c(t)$ เป็นแรงดันเอาต์พุต และ $v(t)$ เป็นแรงดันอินพุต (7 คะแนน)



เมื่อใช้ KVL สามารถเขียนสมการอนุพันธ์ได้ดังนี้

$$v(t) = RC \frac{dv_c(t)}{dt} + v_c(t)$$

- 1.1) จงหา frequency response, $H(j\omega)$ และ impulse response, $h(t)$ (3 คะแนน)

ภาคผนวก

สูตรที่จำเป็น

$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$	$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$
$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$	$a_k = \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt$
$x[n] = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 n}$	$a_k = \frac{1}{N} \sum_{k=-\infty}^{\infty} x[n] e^{-jk\omega_0 n}$
$H(s) = \int_{-\infty}^{\infty} h(\tau) e^{-s\tau} d\tau$	$H(z) = \sum_{k=-\infty}^{\infty} h[k] z^{-k}$
$H(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt$	$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n] e^{-j\omega n}$
$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$	$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$
$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$	$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

TABLE 3.1 PROPERTIES OF CONTINUOUS-TIME FOURIER SERIES

Property	Section	Periodic Signal	Fourier Series Coefficients
		$x(t)$ } Periodic with period T and $y(t)$ } fundamental frequency $\omega_0 = 2\pi/T$	a b
Linearity	3.5.1	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time Shifting	3.5.2	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
Frequency Shifting		$e^{jM\omega_0 t} x(t) = e^{jM(2\pi/T)t} x(t)$	a_{k-M}
Conjugation	3.5.6	$x^*(t)$	a_{-k}^*
Time Reversal	3.5.3	$x(-t)$	a_{-k}
Time Scaling	3.5.4	$x(at), a > 0$ (periodic with period T/a)	a_k
Periodic Convolution		$\int_{\tau} x(\tau)y(t - \tau)d\tau$	$T a_k b_k$
Multiplication	3.5.5	$x(t)y(t)$	$\sum_{l=-\infty}^{\infty} a_l b_{k-l}$
Differentiation		$\frac{dx(t)}{dt}$	$jk\omega_0 a_k = jk \frac{2\pi}{T} a_k$
Integration		$\int_{-\infty}^{\infty} x(t) dt$ (finite valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{jk\omega_0}\right) a_k = \left(\frac{1}{jk(2\pi/T)}\right) a_k$
Conjugate Symmetry for Real Signals	3.5.6	$x(t)$ real	$a_k = a_{-k}^*$ $\Re\{a_k\} = \Re\{a_{-k}\}$ $\Im\{a_k\} = -\Im\{a_{-k}\}$ $ a_k = a_{-k} $ $\angle a_k = -\angle a_{-k}$
Real and Even Signals	3.5.6	$x(t)$ real and even	a_k real and even
Real and Odd Signals	3.5.6	$x(t)$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals		$\begin{cases} x_e(t) = \mathcal{E}\{x(t)\} & [x(t) \text{ real}] \\ x_o(t) = \mathcal{O}\{x(t)\} & [x(t) \text{ real}] \end{cases}$	$\begin{cases} \Re\{a_k\} \\ \Im\{a_k\} \end{cases}$

Parseval's Relation for Periodic Signals

$$\frac{1}{T} \int_T |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |a_k|^2$$

TABLE 3.2 PROPERTIES OF DISCRETE-TIME FOURIER SERIES

Property	Periodic Signal	Fourier Series Coefficients
	$x[n]$ } Periodic with period N and $y[n]$ } fundamental frequency $\omega_0 = 2\pi/N$	a_k } Periodic with b_k } period N
Linearity	$Ax[n] + By[n]$	$Aa_k + Bb_k$
Time Shifting	$x[n - n_0]$	$a_k e^{-jk(2\pi/N)n_0}$
Frequency Shifting	$e^{jM(2\pi/N)n} x[n]$	a_{k-M}
Conjugation	$x^*[n]$	a_{-k}^*
Time Reversal	$x[-n]$	a_{-k}
Time Scaling	$x_{(m)}[n] = \begin{cases} x[n/m], & \text{if } n \text{ is a multiple of } m \\ 0, & \text{if } n \text{ is not a multiple of } m \end{cases}$ (periodic with period mN)	$\frac{1}{m} a_k$ (viewed as periodic with period mN)
Periodic Convolution	$\sum_{r=(N)} x[r]y[n-r]$	$N a_k b_k$
Multiplication	$x[n]y[n]$	$\sum_{l=(N)} a_l b_{k-l}$
First Difference	$x[n] - x[n-1]$	$(1 - e^{-jk(2\pi/N)}) a_k$
Running Sum	$\sum_{k=-\infty}^n x[k]$ (finite valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{(1 - e^{-jk(2\pi/N)})}\right) a_k$
Conjugate Symmetry for Real Signals	$x[n]$ real	$a_k = a_{-k}^*$ $\Re\{a_k\} = \Re\{a_{-k}\}$ $\Im\{a_k\} = -\Im\{a_{-k}\}$ $ a_k = a_{-k} $ $\angle a_k = -\angle a_{-k}$
Real and Even Signals	$x[n]$ real and even	a_k real and even
Real and Odd Signals	$x[n]$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals	$\begin{cases} x_e[n] = \mathcal{E}\{x[n]\} & [x[n] \text{ real}] \\ x_o[n] = \mathcal{O}\{x[n]\} & [x[n] \text{ real}] \end{cases}$	$\begin{cases} \Re\{a_k\} \\ \Im\{a_k\} \end{cases}$

Parseval's Relation for Periodic Signals

$$\frac{1}{N} \sum_{n=(N)} |x[n]|^2 = \sum_{k=(N)} |a_k|^2$$

TABLE 4.1 PROPERTIES OF THE FOURIER TRANSFORM

Section	Property	Aperiodic signal	Fourier transform
		$x(t)$	$X(j\omega)$
		$y(t)$	$Y(j\omega)$

4.3.1	Linearity	$ax(t) + by(t)$	$aX(j\omega) + bY(j\omega)$
4.3.2	Time Shifting	$x(t - t_0)$	$e^{-j\omega t_0} X(j\omega)$
4.3.6	Frequency Shifting	$e^{j\omega_0 t} x(t)$	$X(j(\omega - \omega_0))$
4.3.3	Conjugation	$x^*(t)$	$X^*(-j\omega)$
4.3.5	Time Reversal	$x(-t)$	$X(-j\omega)$
4.3.5	Time and Frequency Scaling	$x(at)$	$\frac{1}{ a } X\left(\frac{j\omega}{a}\right)$
4.4	Convolution	$x(t) * y(t)$	$X(j\omega)Y(j\omega)$
4.5	Multiplication	$x(t)y(t)$	$\frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\theta)Y(j(\omega - \theta))d\theta$
4.3.4	Differentiation in Time	$\frac{d}{dt} x(t)$	$j\omega X(j\omega)$
4.3.4	Integration	$\int_{-\infty}^t x(t)dt$	$\frac{1}{j\omega} X(j\omega) + \pi X(0)\delta(\omega)$
4.3.6	Differentiation in Frequency	$tx(t)$	$j \frac{d}{d\omega} X(j\omega)$
4.3.3	Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} X(j\omega) = X^*(-j\omega) \\ \Re\{X(j\omega)\} = \Re\{X(-j\omega)\} \\ \Im\{X(j\omega)\} = -\Im\{X(-j\omega)\} \\ X(j\omega) = X(-j\omega) \\ \angle X(j\omega) = -\angle X(-j\omega) \end{cases}$
4.3.3	Symmetry for Real and Even Signals	$x(t)$ real and even	$X(j\omega)$ real and even
4.3.3	Symmetry for Real and Odd Signals	$x(t)$ real and odd	$X(j\omega)$ purely imaginary and odd
4.3.3	Even-Odd Decomposition for Real Signals	$x_e(t) = \mathcal{E}\{x(t)\}$ [$x(t)$ real] $x_o(t) = \mathcal{O}\{x(t)\}$ [$x(t)$ real]	$\Re\{X(j\omega)\}$ $j\Im\{X(j\omega)\}$

4.3.7	Parseval's Relation for Aperiodic Signals		
		$\int_{-\infty}^{+\infty} x(t) ^2 dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) ^2 d\omega$	

TABLE 4.2 BASIC FOURIER TRANSFORM PAIRS

Signal	Fourier transform	Fourier series coefficients (if periodic)
$\sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - k\omega_0)$	a_k
$e^{j\omega_0 t}$	$2\pi \delta(\omega - \omega_0)$	$a_1 = 1$ $a_k = 0$, otherwise
$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	$a_1 = a_{-1} = \frac{1}{2}$ $a_k = 0$, otherwise
$\sin \omega_0 t$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$	$a_1 = -a_{-1} = \frac{1}{2j}$ $a_k = 0$, otherwise
$x(t) = 1$	$2\pi \delta(\omega)$	$a_0 = 1$, $a_k = 0$, $k \neq 0$ (it is the Fourier series representation for any choice of $T > 0$)
Periodic square wave		
$x(t) = \begin{cases} 1, & t < T_1 \\ 0, & T_1 < t \leq \frac{T}{2} \end{cases}$ and $x(t + T) = x(t)$	$\sum_{k=-\infty}^{+\infty} \frac{2 \sin k\omega_0 T_1}{k} \delta(\omega - k\omega_0)$	$\frac{\omega_0 T_1}{\pi} \operatorname{sinc}\left(\frac{k\omega_0 T_1}{\pi}\right) = \frac{\sin k\omega_0 T_1}{k\pi}$
$\sum_{n=-\infty}^{+\infty} \delta(t - nT)$	$\frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$	$a_k = \frac{1}{T}$ for all k
$x(t) \begin{cases} 1, & t < T_1 \\ 0, & t > T_1 \end{cases}$	$\frac{2 \sin \omega T_1}{\omega}$	--
$\frac{\sin Wt}{\pi t}$	$X(j\omega) = \begin{cases} 1, & \omega < W \\ 0, & \omega > W \end{cases}$	--
$\delta(t)$	1	--
$u(t)$	$\frac{1}{j\omega} + \pi \delta(\omega)$	--
$\delta(t - t_0)$	$e^{-j\omega t_0}$	--
$e^{-at} u(t), \Re\{a\} > 0$	$\frac{1}{a + j\omega}$	--
$te^{-at} u(t), \Re\{a\} > 0$	$\frac{1}{(a + j\omega)^2}$	--
$\frac{t^{n-1}}{(n-1)!} e^{-at} u(t), \Re\{a\} > 0$	$\frac{1}{(a + j\omega)^n}$	--

TABLE 5.1 PROPERTIES OF THE DISCRETE-TIME FOURIER TRANSFORM

Section	Property	Aperiodic Signal	Fourier Transform
		$x[n]$	$X(e^{j\omega})$ } periodic with
		$y[n]$	$Y(e^{j\omega})$ } period 2π
5.3.2	Linearity	$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
5.3.3	Time Shifting	$x[n - n_0]$	$e^{-jn_0\omega} X(e^{j\omega})$
5.3.3	Frequency Shifting	$e^{jn_0n} x[n]$	$X(e^{j(\omega - \omega_0)})$
5.3.4	Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$
5.3.6	Time Reversal	$x[-n]$	$X(e^{-j\omega})$
5.3.7	Time Expansion	$x_{(k)}[n] = \begin{cases} x[n/k], & \text{if } n = \text{multiple of } k \\ 0, & \text{if } n \neq \text{multiple of } k \end{cases}$	$X(e^{jk\omega})$
5.4	Convolution	$x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
5.5	Multiplication	$x[n]y[n]$	$\frac{1}{2\pi} \int_{2\pi} X(e^{j\theta})Y(e^{j(\omega - \theta)})d\theta$
5.3.5	Differencing in Time	$x[n] - x[n - 1]$	$(1 - e^{-j\omega})X(e^{j\omega})$
5.3.5	Accumulation	$\sum_{k=-\infty}^n x[k]$	$\frac{1}{1 - e^{-j\omega}} X(e^{j\omega})$ $+ \pi X(e^{j0}) \sum_{k=-\infty}^{+\infty} \delta(\omega - 2\pi k)$
5.3.8	Differentiation in Frequency	$nx[n]$	$j \frac{dX(e^{j\omega})}{d\omega}$
5.3.4	Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} X(e^{j\omega}) = X^*(e^{-j\omega}) \\ \Re\{X(e^{j\omega})\} = \Re\{X(e^{-j\omega})\} \\ \Im\{X(e^{j\omega})\} = -\Im\{X(e^{-j\omega})\} \\ X(e^{j\omega}) = X(e^{-j\omega}) \\ \angle X(e^{j\omega}) = -\angle X(e^{-j\omega}) \end{cases}$
5.3.4	Symmetry for Real, Even Signals	$x[n]$ real and even	$X(e^{j\omega})$ real and even
5.3.4	Symmetry for Real, Odd Signals	$x[n]$ real and odd	$X(e^{j\omega})$ purely imaginary and odd
5.3.4	Even-odd Decomposition of Real Signals	$x_e[n] = \text{Ev}\{x[n]\}$ [$x[n]$ real] $x_o[n] = \text{Od}\{x[n]\}$ [$x[n]$ real]	$\Re\{X(e^{j\omega})\}$ $j\Im\{X(e^{j\omega})\}$
5.3.9	Parseval's Relation for Aperiodic Signals		
		$\sum_{n=-\infty}^{+\infty} x[n] ^2 = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega}) ^2 d\omega$	

TABLE 5.2 BASIC DISCRETE-TIME FOURIER TRANSFORM PAIRS

Signal	Fourier Transform	Fourier Series Coefficients (if periodic)
$\sum_{k=(N)} a_k e^{jk(2n/N)n}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$	a_k
$e^{j\omega_0 n}$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega - \omega_0 - 2\pi l)$	(a) $\omega_0 = \frac{2\pi m}{N}$ $a_k = \begin{cases} 1, & k = m, m \pm N, m \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational \Rightarrow The signal is aperiodic
$\cos \omega_0 n$	$\pi \sum_{l=-\infty}^{+\infty} \{\delta(\omega - \omega_0 - 2\pi l) + \delta(\omega + \omega_0 - 2\pi l)\}$	(a) $\omega_0 = \frac{2\pi m}{N}$ $a_k = \begin{cases} \frac{1}{2}, & k = \pm m, \pm m \pm N, \pm m \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational \Rightarrow The signal is aperiodic
$\sin \omega_0 n$	$\frac{\pi}{j} \sum_{l=-\infty}^{+\infty} \{\delta(\omega - \omega_0 - 2\pi l) - \delta(\omega + \omega_0 - 2\pi l)\}$	(a) $\omega_0 = \frac{2\pi r}{N}$ $a_k = \begin{cases} \frac{1}{2j}, & k = r, r \pm N, r \pm 2N, \dots \\ -\frac{1}{2j}, & k = -r, -r \pm N, -r \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational \Rightarrow The signal is aperiodic
$x[n] = 1$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega - 2\pi l)$	$a_k = \begin{cases} 1, & k = 0, \pm N, \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$
Periodic square wave $x[n] = \begin{cases} 1, & n \leq N_1 \\ 0, & N_1 < n \leq N/2 \end{cases}$ and $x[n + N] = x[n]$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \frac{\sin[(2\pi k/N)(N_1 + \frac{1}{2})]}{N \sin[2\pi k/2N]}, k \neq 0, \pm N, \pm 2N, \dots$ $a_k = \frac{2N_1 + 1}{N}, k = 0, \pm N, \pm 2N, \dots$
$\sum_{k=-\infty}^{+\infty} \delta[n - kN]$	$\frac{2\pi}{N} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \frac{1}{N}$ for all k
$a^n u[n], a < 1$	$\frac{1}{1 - ae^{-j\omega}}$	—
$x[n] = \begin{cases} 1, & n \leq N_1 \\ 0, & n > N_1 \end{cases}$	$\frac{\sin[\omega(N_1 + \frac{1}{2})]}{\sin(\omega/2)}$	—
$\frac{\sin Wn}{\pi n} = \frac{W}{\pi} \text{sinc}\left(\frac{Wn}{\pi}\right)$ $0 < W < \pi$	$X(\omega) = \begin{cases} 1, & 0 \leq \omega \leq W \\ 0, & W < \omega \leq \pi \end{cases}$ $X(\omega)$ periodic with period 2π	—
$\delta[n]$	1	—
$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{+\infty} \pi \delta(\omega - 2\pi k)$	—
$\delta[n - n_0]$	$e^{-j\omega n_0}$	—
$(n+1)a^n u[n], a < 1$	$\frac{1}{(1 - ae^{-j\omega})^2}$	—
$\frac{(n+r-1)!}{n!(r-1)!} a^n u[n], a < 1$	$\frac{1}{(1 - ae^{-j\omega})^r}$	—