# มหาวิทยาลัยสงขลานครินทร์
## คณะวิศวกรรมศาสตร์

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| **การสอบกลางภาค ประจำภาคการศึกษาที่** 2 | **ปีการศึกษา** 2554 |
| **สอบวันที่** 20 **ธันวาคม** 2554 | **เวลา** 9:00-12:00 **น** |
| **วิชา** 220-622 Groundwater Flow Modeling | **ห้องสอบ** Robot |

ข้อกำหนด

1. ข้อสอบ มี 4 ข้อ จำนวน 5 หน้า คะแนนเต็ม 200 คะแนน ให้ทำทุกข้อ
2. อนุญาตให้ Lecture note เข้าห้องสอบได้
3. ให้นำเครื่องคิดเลขทุกชนิดเข้าห้องสอบได้
4. ให้นักศึกษาตอบในสมุดคำตอบ

ชื่อ.......................................................................................รหัส.................................

| ข้อ | คะแนนเต็ม | คะแนนที่ได้ |
|---|---|---|
| 1 | 40 | |
| 2 | 60 | |
| 3 | 50 | |
| 4 | 50 | |
| Total | 200 | |

ออกข้อสอบโดย อ. ธนิต เฉลิมยานนท์

14 ธ.ค. 2554

ชื่อ..................................................................รหัส....................

1. Definitions

Briefly define and explain terms shown in each question. Each definition is worth 5 points.

1.1 Central difference approximation

1.2 Laplace's equation

1.3 Specific storage

1.4 Successive Over-Relaxation

1.5 Explicit finite difference approximation

1.6 Conceptual model

1.7 Dupuit's assumptions

1.8 Derichlet and Neumann conditions

2. Equations and Finite Difference Approximation

2.1 (10 points) Write the governing equation for two-dimensional, steady state flow in homogeneous, but anisotropic unconfined aquifer under Dupuit assumptions. Include a source term to simulate recharge to the aquifer.

2.2 (15 points) The mass balance equation for flow through an representative elementary volume is

$$div\, q = 0$$

(a) Write this equation in three dimensions using partial differentials.

(b) Write Darcy's law in vector notation and combine it with $div\, q = 0$ to derive the general governing equation groundwater flow. What is the name of the equation just derived.

2.3 (20 points) Write a finite difference expression for the following equation, where Dx, Dy, v are constant but $\Delta x \neq \Delta y$

$$D_x \frac{\partial^2 c}{\partial x^2} + D_y \frac{\partial^2 c}{\partial y^2} - v \frac{\partial c}{\partial x} = 0$$

Also write finite difference formula that $c_{i,j}$ can be computed for the specific case where $D_x = D_y = D$ and $\Delta x = \Delta y = a$

2.4 (15 points) From conservation of mass, derive governing equation of transient flow for 2-D unconfined aquifer with sink/source term and also write explicit finite difference approximation for this equation.

3. System Conceptualization

3.1 (25 points) Develop a mathematical model (governing equation and boundary conditions) for the flow system illustrated in the sketch below. Indicate the locations of the boundary conditions mathematically rather than by labeling a sketch. Note that R and q must be treated as boundary conditions.

A sketch shows a profile view of a heterogeneous, anisotropic unconfined aquifer under steady state conditions. The q is outflow flux.

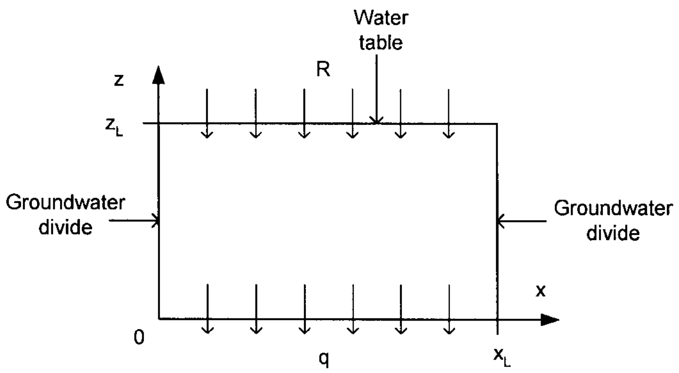


Fig.1 An aquifer system for problem 3.1

3.2 (25 points) The unconfined island recharge problem, with addition of a newly installed pumping well at location B as illustrated in the sketch below. Sea water level is 100 m from datum.

(a) Write the governing equation that you would use to solve the transient problem to predict the decline in head as a result of pumping.

(b) How would you generate the initial conditions in term of head for transient problem.

(c) Write the equation for computing a water balance in terms of inflow, outflow and storage for this transient problem.

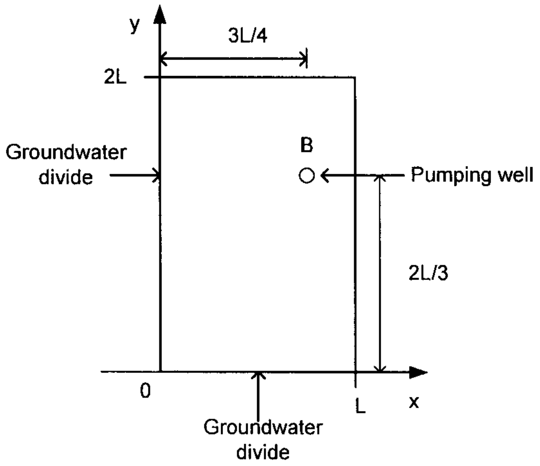


Fig. 2 Island recharge problem for problem 3.2

4. Head and Flux Determination

4.1 (25 points) Consider the 6 nodes shown below. Suppose these nodes are the 6 nodes in the upper left hand corner of a two-dimensional horizontal finite difference grid of a confined aquifer. The nodes in the top row are specified head cells and the nodes in the bottom row are active cells in a steady-state simulation. The numbers in each cell are the head values (block-centered) . Suppose we wish to switch from specified head to specified flux boundary conditions. Calculate the values of the flux ($m^3$/day) that you would assign to boundary cell A and B. Be sure to indicate the sign of the flux, where (+) indicates inflow to the model and (-) indicates withdrawal of water from the model. Assume that the aquifer is isotropic and transmissivity is 300 $m^2$/day, and the nodes are equally spaced such that $\Delta x = \Delta y = 100$ m.

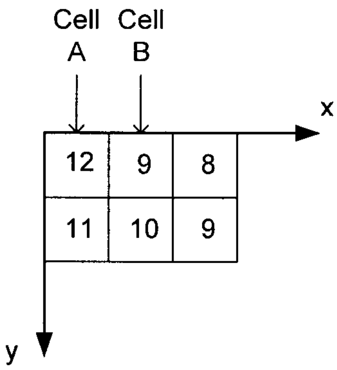


Fig.3 Domain for flux calculation (numbers in the cell indicating the head at each cell)

4.2 (25 points) A groundwater flow domain where observed heads (in meters) are shown for each node. Suppose this domain is a 2-D steady-state confined aquifer where Laplace's equation is applied. Calculate the heads at node A and B.

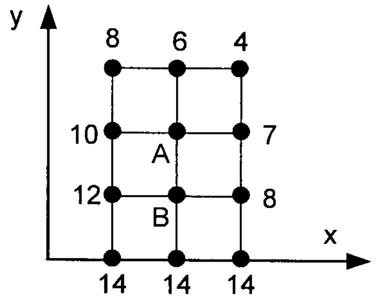


Fig. 4 A confined aquifer domain