มหาวิทยาลัยสงขลานครินทร์

คณะวิศวกรรมศาสตร์

สอบกลางภาค ประจำภาคการศึกษา 2 ปีการศึกษา 2555

วันที่ 19 ธันวาคม 2555 9.00 — 12.00

A 400

วิชา CE 220-504: Introduction to Finite Element Method

ชื่อ-สกุล...

รหัส...........................

คำชี้แจง

1.ข้อสอบทั้งหมดมี 5 ข้อ คะแนนรวม 150 คะแนน ดังแสดงในตารางข้างล่าง

2.ข้อสอบมีทั้งหมด 4 หน้า (รวมปก) ผู้สอบต้องตรวจสอบว่ามีครบทุกหน้าหรือไม่ (ก่อนลงมือทำ)

3.ให้ทำหมดทุกข้อลงในสมุดคำตอบ

4.อนุญาตให้ใช้เครื่องคิดเลขได้ทุกชนิด

5.ห้ามหยิบ หรือยืมสิ่งของใดๆ ของผู้อื่นในห้องสอบ

6. ***Open Books***

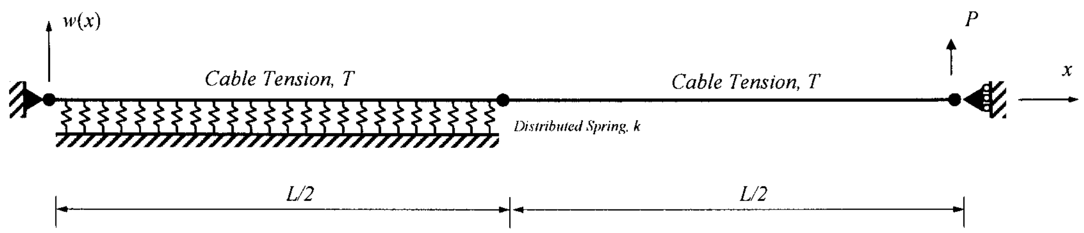
7. ***GOOD LUCK***

ตารางคะแนน

ข้อที่	คะแนนเต็ม	ได้
1	30	
2	30	
3	30	
4	30	
5	30	
รวม	150	

Problem 1 (30 Points)

A prestressed cable with tension, T is partially supported by a distributed stiffness of k as shown below. The load is a vertical force P acting at the end free to translate vertically. Axial deformation of the cable is neglected.



Two boundary conditions of this problem are:

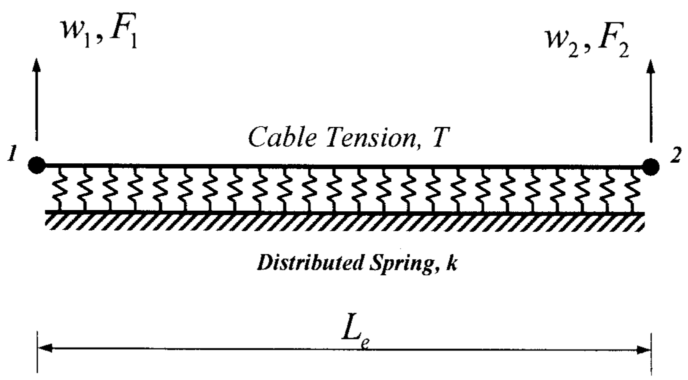
Essential Boundary Condition: $w(0) = 0$

Natural Boundary Condition: $Tw'(L) = P$

Assume that $k = T / L^2$ and $P = T = L = 1$

Discretize this cable by two linear cable finite elements and determine its nodal displacements as well as the vertical force at the left end.

Given: the element stiffness equation of a linear cable element with lateral support is:



$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \left(\frac{T}{L_e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{kL_e}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \right) \begin{Bmatrix} w_1 \\ w_2 \end{Bmatrix}$$

Problem 2 (30 Points)

The governing differential equation for the torsion of a thin-walled section with warping restraint can be written as follows:

$$EJ_w \frac{d^4\phi(x)}{dx^4} - GJ_0 \frac{d^2\phi(x)}{dx^2} - t(x) = 0 \ for \ 0 \le x \le L$$

where E is the modulus of elasticity; G is the shear modulus; J_w is the warping constant; J_0 is the torsional constant; t is thickness of the section; and ϕ is the angle through which the cross section rotates. Derive the weak form, functional form (if possible), and indicate appropriate essential and natural boundary conditions.

Problem 3 (30 Points)

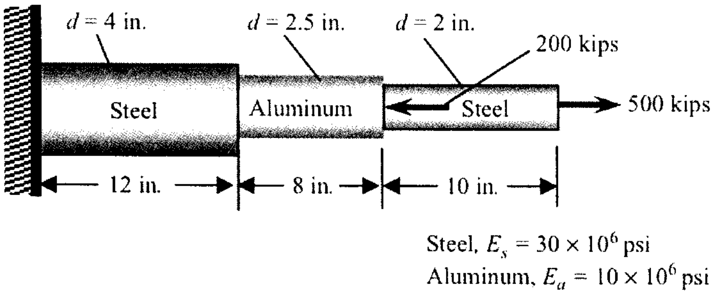
The governing differential equation is of the form:

$$-\frac{d}{dx}\left[EA\frac{du}{dx}\right] = 0; 0 < x < L$$

For the minimum number of linear elements, give

(a) the boundary conditions on the nodal variables (primary as well as secondary)

(b) the final condensed finite element equations for the unknowns



Problem 4 (30 Points)

Solve the following differential equation for the natural (or Neumann) boundary conditions:

$$-\frac{d^2u}{dx^2} - u = 0 : 0 < x < 1$$

Natural Boundary Conditions:

$$\left(\frac{du}{dx}\right)_{x=0} = 1 \text{ and } \left(\frac{du}{dx}\right)_{x=1} = 0$$

Use the uniform mesh of two linear finite elements to solve the problem.

Problem 5 (30 Points)

For the structure shown in Figure below, determine the forces and elongation in rods AB and CD. Each rod has a cross-sectional area A of 0.03 in^2 and modulus of elasticity $E = 30 \times 10^6\, psi$.

Please use ***Penalty Method*** to cope with the kinematics constraint.