$\qquad$ ID


Prince of Songkla University
Faculty of Engineering

Name $\qquad$ ID $\qquad$

Direction:

1. All types of calculator and dictionary are permitted.
2. There are totally 5 problems.
3. One sheet of hand-written A4 paper is allowed. No photocopy!!

Perapong Tekasakul Kittinan Maliwan

Instructors

| Problem <br> No. | Full score | Your mark |
| :---: | :---: | :---: |
| 1 | 15 |  |
| 2 | 10 |  |
| 3 | 15 |  |
| 4 | 28 |  |
| 5 | 12 |  |
| Total | $\mathbf{8 0}$ |  |

1. Determine the root of $\sin x=x^{3}$, where $x$ is in radian, using bisection method to determine the root to $\varepsilon_{a}=2 \%$. Employ initial guesses of $x_{l}=0.5$ and $x_{u}=1.0$. ( 15 points)

| Iter. | $\mathrm{x}_{1}$ | $\mathrm{x}_{\mathrm{u}}$ | $\mathrm{x}_{\mathrm{r}}$ | $\varepsilon_{\mathrm{a}}$ |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

2. Give the system of equations (10 points)

$$
\begin{aligned}
& x_{1}+x_{2}-x_{3}=-3 \\
& 6 x_{1}+2 x_{2}+2 x_{3}=2 \\
& -3 x_{1}+4 x_{2}+x_{3}=1
\end{aligned}
$$

(a) Use Gauss elimination to solve for the $x^{\prime} s$
(b) Substitute your results back into the original equation to check your solution
3. Given the data

| $x$ | 1 | 2 | 3 | 5 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 3 | 6 | 19 | 99 | 291 | 444 |

Calculate $f(4)$ using Lagrange Polynomials of orders 1, 2, 3. (15 points)
4. (28 points)
4.1 The function $f(x)=2 e^{-1.5 x}$ can be used to generate the following table of unequally spaced data:

| $x$ | 0 | 0.05 | 0.15 | 0.25 | 0.35 | 0.475 | 0.6 |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $f(x)$ | 2 | 1.8555 | 1.5970 | 1.3746 | 1.1831 | 0.9808 | 0.8131 |

Evaluate the integral from $\mathrm{a}=0$ to $\mathrm{b}=0.6$ using
(a) analytical means
(b) the trapezoidal rule, and
(c) the best combination of the trapezoidal and Simpson's rules

For (b) and (c), compute the percent relative error $\left(\varepsilon_{\mathrm{t}}\right)$. ( 10 points)
(a)
(b)
(c)
$\qquad$
4.2 Evaluate

$$
\int_{0}^{3} x e^{x} d x
$$

Using
(a) analytical means
(b) order of $h^{8}$ Romberg integration
(c) four-point Gauss-Legendre formula

For (b) and (c), compute the percent relative error $\left(\varepsilon_{\mathrm{t}}\right)$. (18 points)
(a)
(b)

| n | $\mathrm{O}\left(\mathrm{h}^{2}\right)$ | $\mathrm{O}\left(\mathrm{h}^{4}\right)$ | $\mathrm{O}\left(\mathrm{h}^{6}\right)$ | $\mathrm{O}\left(\mathrm{h}^{8}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |

(c)
5. The following data was collected for the distance traveled versus time for a rocket:

| $t(s)$ | 0 | 25 | 50 | 75 | 100 | 125 |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $y(k m)$ | 0 | 32 | 58 | 78 | 92 | 100 |

Use the best numerical method available of accuracy $O\left(h^{2}\right)$ to estimate the rocket's velocity and acceleration at each time. ( 12 points)

| $t(\mathrm{~s})$ | $y(\mathrm{~km})$ | $v(\mathrm{~m} / \mathrm{s})$ | $a\left(\mathrm{~m} / \mathrm{s}^{2}\right)$ |
| :---: | :---: | :---: | :---: |
| 0 | 0 |  |  |
| 25 | 32 |  |  |
| 50 | 58 |  |  |
| 75 | 78 |  |  |
| 100 | 92 |  |  |
| 125 | 100 |  |  |

