PRINCE OF SONGKLA UNIVERSITY

FACULTY OF ENGINEERING

Department of Computer Engineering

Final Examination: Semester 2

Academic Year: 2013-2014

Date: 4th March, 2014

Time: 9:00 – 12:00 (**3 hours**)

Subject Numbers: 242-213

Room: Robot Head

Subject Title: Discrete Mathematics

Lecturer: Aj. Andrew Davison

Exam Duration: 3 hours

This paper has 9 questions, in 4 pages.

Authorized Materials:

- Writing instruments (e.g. pens, pencils).
- Books (e.g. dictionaries) and calculators are **not** permitted.

Instructions to Students:

- *Answer questions in English.* Perfect English is **not** required.
- Attempt all questions.
- Write your answers in an answer book.
- Start your answer to each question on a new page
- Clearly number your answers.
- Any unreadable parts will be considered wrong.
- When writing programs, use good layout, and short comments; marks will not be deducted for minor syntax errors.
- The marks for each part of a question are given in brackets (…).

Question 1 (20 minutes; 20 marks)

Prove De Morgan's first law **twice**: once using the set builder notation **and again** using membership tables. The law:

$$\overline{A \cup B} = \bar{A} \cap \bar{B}$$

Question 2 (20 minutes; 20 marks)

Explain the following function mappings:

a) injective (5)

b) surjective (5)

c) bijective (5)

d) inverse (5)

Explain each of your answers in words, with a diagram, and with a small function example.

Question 3 (20 minutes; 20 marks)

How many bit strings of length 10 either begin with three 0's or end with two 0's?

Explain all your working, not just the final answer.

Question 4 (20 minutes; 20 marks)

The name of a variable in Java is a string of between 1 and 65,535 characters in length. Each character can be an uppercase or a lowercase letter, a dollar sign, an underscore ('_'), or a digit, except that the first character must not be a digit.

Determine the number of different variable names in Java.

Explain all your working, not just the final answer.

Hint: use the geometric series in your answer:

Question 5 (30 minutes; 30 marks)

Use induction to show that each of the following equations is true:

a) $3 + 3\cdot 5 + 3\cdot 5^2 + \cdots + 3\cdot 5^n = 3*(5^{(n+1)} - 1) / 4$, when $n \geq 0$ (10)

b) $2 - 2\cdot 7 + 2\cdot 7^2 - \cdots + 2(-7)^n = (1 - (-7)^{(n+1)}) / 4$, when $n \geq 0$ (10)

c) $3^n < n!$, when $n > 6$ (Math help: $3^6 = 729$, $6! = 720$) (10)

Question 6

(10 minutes; 10 marks)

Show that $x^4 + 9x^3 + 4x+7$ is $O(x^4)$. You must use witnesses to explain your answer.

Question 7

(15 minutes; 15 marks)

The conventional algorithm for evaluating a polynomial $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ at x = c can be expressed in pseudocode as:

```
double polynomial(double c, double a0, double a1, ..., double an)
{
  power = 1
  y = a0
  for i = 1 to n {
    power = power*c
    y = y + ai*power
  }
  return y    //  the value of the polynomial at x = c
}
```

a) Evaluate $3x^2 + x + 1$ at $x = 2$ by working through each step of the algorithm showing the values assigned at each assignment step. (10)

b) Exactly how many multiplications and additions are used to evaluate a polynomial of degree n at x = c? Do not count additions used to increment the loop variable. Show all your working. (5)

Question 8

(15 minutes; 15 marks)

There is a more efficient algorithm (in terms of the number of multiplications and additions used) for evaluating polynomials than the algorithm in the previous question. It is called *Horner's* method.

The following pseudocode shows how to use this method to find the value of $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ at x = c.

```
procedure horner(double c, double a0, double a1, ..., double an)
{
  y = an
  for i = 1 to n
    y = y*c + an-i
  return y
}
```

a) Evaluate $3x^2 + x + 1$ at $x = 2$ by working through each step of the algorithm showing the values assigned at each assignment step. (10)

b) Exactly how many multiplications and additions are used by this algorithm to evaluate a polynomial of degreen at x = c? Do not count additions used to increment the loop variable. Show all your working. (5)

Question 9

(30 minutes; 30 marks)

a) Write a *recursive* C function `largestElem()` that takes **only** a `LIST` argument as input, and returns the *largest* element in the list. Assume that the list contains only positive integers. If the list is empty, the function returns -1. (10)

b) Write an *iterative* C function (i.e. one using a loop or loops) which does the same task as in (a). Do **not** use recursion. (10)

c) Compare the functions of part (a) and (b), and say in words which is more **space** efficient. Explain your decision. (10)

Hint: efficiency in this case means the amount of memory used to store data. Do **not** use big-Oh notation.

--- End of Examination ---