PRINCE OF SONGKLA UNIVERSITY

FACULTY OF ENGINEERING

Department of Computer Engineering

Midterm Examination: Semester 2

Academic Year: 2014-2015

Date: 16th March, 2015

Time: 9:00 – **11:00 (2 hours)**

Subject Numbers: 242-213 **and** 241-303

Room: Robot Head **and** ~~A200~~ A400

Subject Title: Discrete Mathematics

Lecturer: Aj. Andrew Davison

Exam Duration: 2 hours

This paper has 8 questions, in 3 pages.

Authorized Materials:

- Writing instruments (e.g. pens, pencils).
- Books (e.g. dictionaries) and calculators are **not** permitted.

Instructions to Students:

- *Answer questions in English.* Perfect English is **not** required.
- Attempt all questions.
- Write your answers in an answer book.
- Start your answer to each question on a new page
- Clearly number your answers.
- Any unreadable parts will be considered wrong.
- When writing programs, use good layout, and short comments; marks will not be deducted for minor syntax errors.
- The marks for each part of a question are given in brackets (...).

Question 1
(10 minutes; 10 marks)

Show that $(p \land q) \rightarrow (p \lor q)$ is a tautology by using rules of inference. Do **not** use truth tables.

Question 2
(20 minutes; 20 marks)

Consider these statements: the first three are assumptions and the fourth is a conclusion.

- "All hummingbirds are brightly colored."
- "No large birds live on honey."
- "Birds that do not live on honey are not brightly colored."
- "Hummingbirds are not large."

Let P(x), Q(x), R(x), and S(x) be the statements "x is a hummingbird," "x is large," "x lives on honey," and "x is brightly colored" . Assume that the domain consists of all birds.

a) Express the statements using quantifiers and P(x), Q(x), R(x), and S(x). (8)

b) Show that the conclusion is provable from the three assumptions. (12)

Question 3
(15 minutes; 15 marks)

Prove by **contrapositive** that if $n = a*b$, where a and b are positive integers, then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$.

Question 4
(15 minutes; 15 marks)

Prove by **contradiction** that if n is an integer and n^3+5 is odd, then n is even.

Question 5
(20 minutes; 20 marks)

Use a **proof by cases** to show that $|xy| = |x||y|$, where x and y are real numbers. $|a|$ is the **absolute** value of a. $|a|$ equals a when $a \geq 0$ and $-a$ when $a < 0$.

Question 6
(15 minutes; 15 marks)

For each of the following relations on the set {1,2,3,4}, state whether the relation is **reflexive, symmetric, transitive**, or none of those properties. Note: some of the relations satisfy more than one property.

Do not write a single-word answer, explain each answer in **words and diagrams**.

a) { (2,2), (2,3), (2,4), (3,2), (3,3), (3,4) }

b) { (1,1), (1,2), (2,1), (2,2), (3,3), (4,4) }

c) { (2,4), (4,2) }

d) { (1,1), (2,2), (3,3), (4,4) }

e) { (1,3), (1,4), (2,3), (2,4), (3,1), (3,4) }

Question 7
(10 minutes; 10 marks)

Determine whether each of these functions is **bijective** from **R** to **R**.

Do not write a single-word answer, explain each answer in **words and diagrams**.

a) $f(x) = 2x+1$

b) $f(x) = x^2+1$

c) $f(x) = x^3$

d) $f(x) = (x^2+1) / (x^2+2)$

Question 8
(15 minutes; 15 marks)

Each user on a computer system has a password, which is six to eight characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit.

How many possible passwords are there?

Do not only write a single numerical answer, also explain the answer in **words**.

Express numerical values in power form (e.g. 9^2), rather than as integers (81).

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