**คณะวิศวกรรมศาสตร์**

**มหาวิทยาลัยสงขลานครินทร์**

การสอบปลายภาค ประจำภาคการศึกษาที่ 1 ประจำปีการศึกษา 2558

วันที่ 8 ธันวาคม 2558 เวลา 13.30-16.30 น.

วิชา 215-612 Finite Element Method ห้อง A 401

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<u>คำสั่ง</u>

1. There are 4 problems
2. This is opened books & Note Examination
3. All books are allowed

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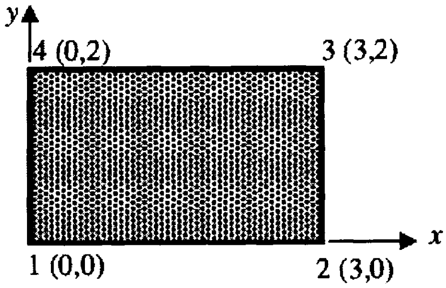
ผู้ออกข้อสอบ

**ทุจริตในการสอบ ปรับขั้นต่ำคือปรับตกในรายวิชาที่ทุจริต และพักการศึกษา 1 ภาคการศึกษา**

1. For a rectangular element shown in the figure, displacements at the four nodes are given by $\{u_1, v_1, u_2, v_2, u_3, v_3, u_4, v_4\} = \{0.0, 0.0, 1.0, 0.0, 2.0, 1.0, 0.0, 2.0\}$. Calculate displacement $(u, v)$ and strain $\varepsilon_{xx}$ at $(x, y) = (2, 1)$.




Where;

$N_1(x) = \frac{1}{A}(x_3 - x)(y_3 - y)$

$N_2(x) = \frac{-1}{A}(x_1 - x)(y_3 - y)$

$N_3(x) = \frac{1}{A}(x_1 - x)(y_1 - y)$

$N_4(x) = \frac{-1}{A}(x_3 - x)(y_1 - y)$

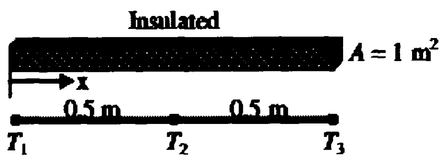
2. In order to solve 1-D steady-state heat transfer problem, one element with 3-nodes is used. The shape functions and the conductivity matrix before applying boundary conditions are given.

$$\begin{cases} N_1(x) = 1 - 3x + 2x^2 \\ N_2(x) = 4x - 4x^2 \\ N_3(x) = -x + 2x^2 \end{cases}, \quad [\mathbf{K}_T] = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 2 \end{bmatrix}$$

(a) When the temperature at node 1 is equal to 40°C and a heat flux of 80 W is input at node 3, calculate the temperature at $x = ¼$ m.

(b) When the temperature at node 1 is equal to 40°C and the convection boundary condition is applied at node 3 with $h = 4$ W/m$^2$/°C, $T^\infty = 100$°C, calculate the temperature at $x = ¼$ m.

(c) Instead of the previous boundary conditions, heat fluxes at nodes 1 and 3 are given as $Q_1$ and $Q_3$, respectively. Can this problem be solved for the nodal temperatures? Explain your answer.

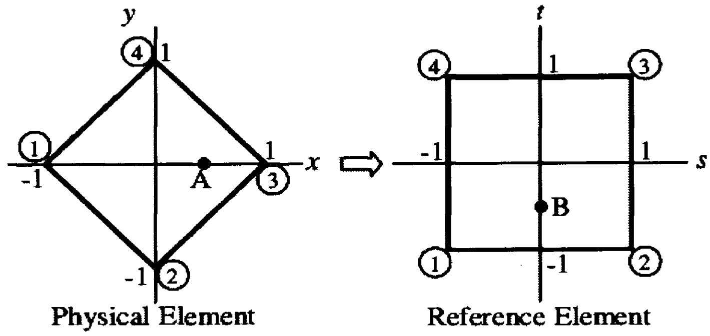


3. The quadrilateral element shown in the figure has the nodal displacements of $\{u_1, v_1, u_2, v_2, u_3, v_3, u_4, v_4\} = \{-1, 0, -1, 0, 0, 1, 0, 1\}$.

(a) Find the $(s, t)$ reference coordinates of point $A$ (0.5, 0) using iso–parametric mapping method.

(b) Calculate the displacement at point $B$ whose reference coordinate is $(s,t)=(0,-0.5)$

(c) Calculate the Jacobian matrix [J] at point $B$.




4. Integrate the following function using one–point and two–point numerical integration (Gauss quadrature). Explain how to integrate it. The exact integral is equal to 2. Compare the accuracy of the numerical integration with the exact one.

$$I = \int_0^{\pi} \sin(x)\, dx$$