มหาวิทยาลัยสงขลานครินทร์
คณะวิศวกรรมศาสตร์

สอบกลางภาค ประจำภาคการศึกษา 2 ปีการศึกษา 2558
วันที่ 29/2/2559 เวลา 9.00 – 12.00
วิชา CE 220-504: Introduction to Finite Element Method ห้องสอบ A 301

ชื่อ-สกุล..
รหัส............................

คำชี้แจง

1.ข้อสอบทั้งหมดมี 5 ข้อ คะแนนรวม 250 คะแนน ดังแสดงในตารางข้างล่าง
2.ข้อสอบมีทั้งหมด 4 หน้า (รวมปก) ผู้สอบต้องตรวจสอบว่ามีครบทุกหน้าหรือไม่ (ก่อนลงมือทำ)
3.ให้ทำหมดทุกข้อลงในสมุดคำตอบ
4.อนุญาตให้ใช้เครื่องคิดเลขได้ทุกชนิด
5.ห้ามหยิบ หรือยืมสิ่งของใดๆ ของผู้อื่นในห้องสอบ
6. *Open Books*
7. *GOOD LUCK*

ตารางคะแนน

ข้อที่	คะแนนเต็ม	ได้
1	50	
2	50	
3	50	
4	50	
5	50	
รวม	250	

Problem 1 (50 Points)

An engineering-analysis problem is formulated in terms of the following ordinary differential equation.

$$\frac{d^2u}{dx^2} + x\frac{du}{dx} - 2u + 1 = 0; \qquad 0 < x < 1$$

with the following boundary conditions:

$$u(0) = 2 \qquad \text{and} \qquad \left.\frac{du}{dx}\right|_{x=1} = u(1)$$

(a) What is the order of the differential equation?

(b) Is the boundary condition at $x = 0$ a natural or essential boundary condition?

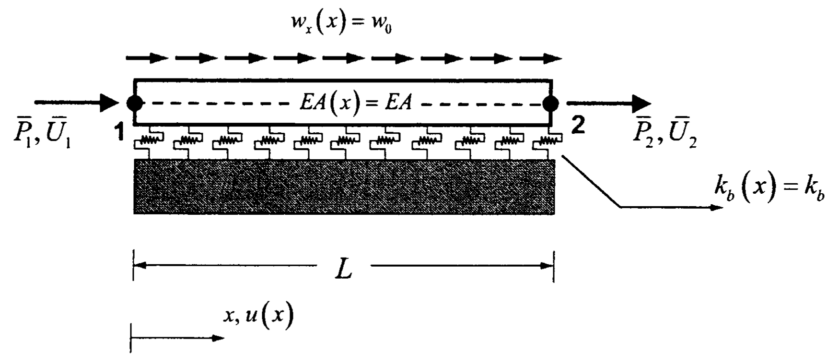
(c) Is the boundary condition at $x = 1$ a natural or essential boundary condition?

(d) Derive a weak form for the problem

(e) With a linear polynomial $u(x) = a_0 + a_1 x$, obtain an approximate solution of the problem

Problem 2 (50 Points)

For the prismatic bar with tangential spring k_b shown in figure below,



(a) Show that the governing differential equation of this bar-spring system is:

$$EA\frac{d^2u}{dx^2} - k_b u + w_0 = 0 \text{ for } 0 < x < L$$

(b) Derive a weak form for this bar-spring system

(c) Show that the total potential energy of this bar-spring system is:

$$\Pi[u]=\frac{1}{2}\int_0^L EA\left(\frac{du}{dx}\right)^2 dx+\frac{1}{2}\int_0^L k_0 u^2 dx-\int_0^L w_0 u dx-\bar{P}_1\bar{U}_1-\bar{P}_2\bar{U}_2$$

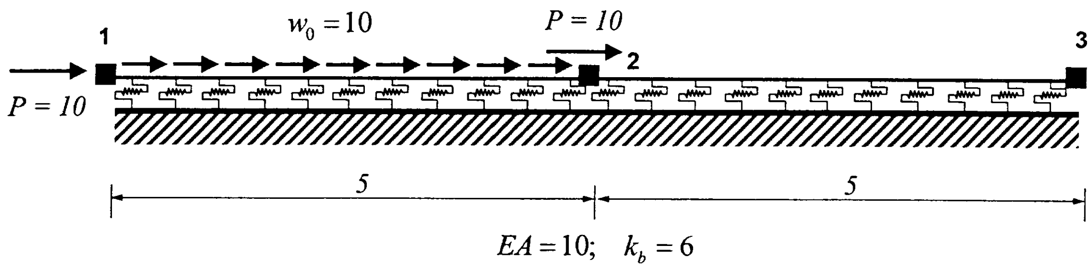
Problem 3 (50 Points)

For the bar-spring system of Problem 2, show that the finite element equation for the two-node bar model is:

$$\left(\frac{EA}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}+\frac{k_b L}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\right)\begin{Bmatrix}\bar{U}_1\\ \bar{U}_2\end{Bmatrix}=\begin{Bmatrix}\bar{P}_1\\ \bar{P}_2\end{Bmatrix}+\begin{Bmatrix}\frac{w_0 L}{2}\\ \frac{w_0 L}{2}\end{Bmatrix}$$

Problem 4 (50 Points)

Two linear bar-spring elements given in Problem 3 are employed to model the pile embedded in soil medium shown below.



You are asked to:

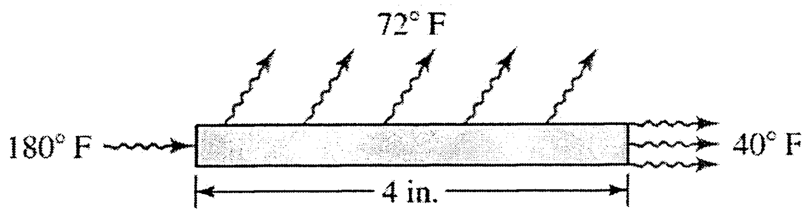
(a) Compute the nodal displacements at nodes 1, 2, and 3

(b) Compute the axial force distribution along the bar length

(c) Verify that the local equilibrium is violated while the global equilibrium is satisfied

Problem 5 (50 Points)

A cylindrical rod that is one of several in a small heat exchange device is shown in Figure below. The left end of the pin is subjected to a constant temperature of 180 F^0 and the right end is in contact with a chilled water bath maintained at constant temperature of 40 F^0. The exterior surface of pin is in contact with moving air at 72 F^0.



The physical data are given as:

$$k = 120 \frac{Btu}{hr - ft - F^0} : Thermal\ Conductivity$$

$$D = 0.5\ in.: Diameter\ of\ Pin$$

$$L = 4\ in.: Length\ of\ Pin$$

$$\beta_{air} = 50 \frac{Btu}{hr - ft^2 - F^0} : Heat\ Transfer\ Coefficient\ of\ Air$$

$$\beta_{water} = 100 \frac{Btu}{hr - ft^2 - F^0} : Heat\ Transfer\ Coefficient\ of\ Water$$

Use a single three-node thermal element to determine the temperature distribution along the rod length.