คณะวิศวกรรมศาสตร์
มหาวิทยาลัยสงขลานครินทร์

ข้อสอบกลางภาค ประจำภาคการศึกษาที่ 1 ประจำปีการศึกษา 2559

วันที่ 15 ตุลาคม 2559 เวลา 13:30-16:30 น.

วิชา 215-333 HEAT TRANSFER ห้อง หัวหุ่น

คำสั่ง

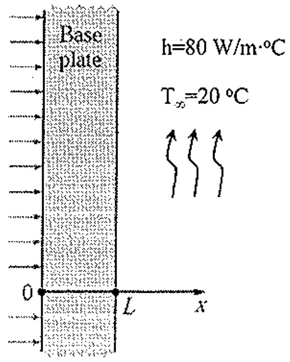
1. ข้อสอบมีทั้งหมด 5 ข้อ 12 หน้า ให้ทำลงในข้อสอบทุกข้อ
2. ใช้ดินสอหรือปากกาทำข้อสอบได้
3. หากกระดาษไม่พอ ให้ทำต่อด้านหลังของข้อสอบได้
4. อนุญาตให้ใช้เครื่องคิดเลขได้
5. ไม่อนุญาตนำเอกสารทุกชนิดเข้าห้องสอบ

อ.มักตาร์ แวหะยี
ผู้ออกข้อสอบ

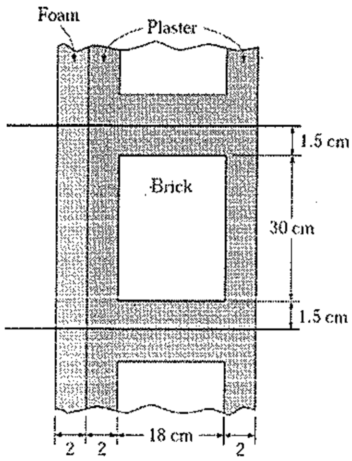
ข้อ	คะแนนเต็ม	คะแนนที่ได้
1	20	
2	20	
3	20	
4	20	
5	20	
รวม	100	

ทุจริตในการสอบ ปรับขั้นต่ำคือปรับตกในรายวิชาที่ทุจริตและพักการศึกษา 1 ภาคการศึกษา

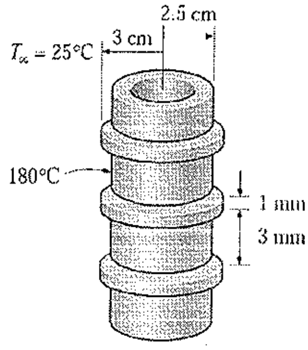
1. Consider the base plate of a 1200-W household iron with a thickness of L=0.5 cm, base area of $A=300\ cm^2$, and thermal conductivity of k=15 W/m·°C. The inner surface of the base plate is subjected to uniform heat flux generated by the resistance heaters inside. When steady operating conditions are reached, the outer surface loses heat to the surroundings at $T_\infty=20^\circ C$ by convention. Taking the convection heat transfer coefficient to be $h=80\ W/m^2\cdot°C$ and disregarding heat loss by radiation, obtain an expression for the variation of temperature in the base plate, and evaluate the temperatures at the inner and the outer surfaces. (20 Marks)



2. A 4-m-high and 6-m-wide wall consists of a long 18-cm × 30-cm cross section of horizontal bricks (k=0.72 W/m·°C) separated by 3-cm-thick plaster layers (k=0.22 W/m·°C). There are also 2-cm-thick plaster layers on each side of the wall, and a 2-cm-thick rigid foam (k=0.026 W/m·°C) on the inner side of the wall. The indoor and the outdoor temperatures are 22°C and -4°C, and the convection heat transfer coefficients on the inner and the outer sides are h_1=10 W/m^2·°C and h_2=20 W/m^2·°C, respectively. Assuming one-dimensional heat transfer and disregarding radiation, determine the rate of heat transfer through the wall. (20 Marks)

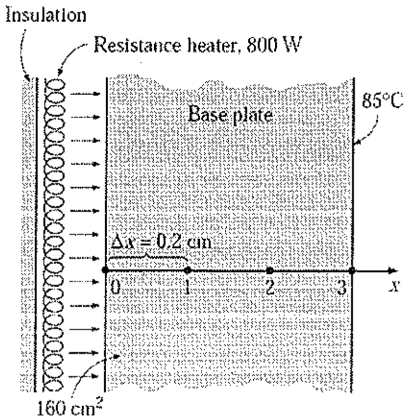


3. Steam in a heating system flows through tubes whose outer diameter is 5 cm and whose walls are maintained at a temperature of 180°C. Circular aluminum alloy 2024-T6 fins (k=186 W/m·°C) of outer diameter 6 cm and constant thickness 1 mm are attached to the tube. The space between the fins is 3 mm, and thus there are 250 fins per meter length of the tube. Heat is transferred to the surrounding air at T=25°C, with a heat transfer coefficient of 40 $W/m^2 \cdot °C$. Determine the increase in heat transfer from the tube per meter of its length as a result of adding fins. (20 Marks)



4. The temperature of a gas stream is to be measured by a thermocouple whose junction can be approximated as a 1-mm-diameter sphere. The properties of the junction are k=35 W/m·°C, density of 8500 kg/m^3, and C_p=320 J/kg·°C, and the convection heat transfer coefficient between the junction and the gas is h=210 W/m^2·°C. Determine how long it will take for the thermocouple to read 99 percent of the initial temperature difference.

5. Consider the base plate of a 800-W household iron having a thickness of L=0.6 cm, base area of A=160 cm2, and thermal conductivity of k └ 20 W/m · °C. The inner surface of the base plate is subjected to uniform heat flux generated by the resistance heaters inside. When steady operating conditions are reached, the outer surface temperature of the plate is measured to be 85°C. Disregarding any heat loss through the upper part of the iron and taking the nodal spacing to be 0.2 cm, (a) obtain the finite difference formulation for the nodes and (b) determine the inner surface temperature of the plate by solving those equations. (20 Marks)



SUMMARY

In this chapter we have studied the heat conduction equation and its solutions. Heat conduction in a medium is said to be *steady* when the temperature does not vary with time and *unsteady* or *transient* when it does. Heat conduction in a medium is said to be *one-dimensional* when conduction is significant in one dimension only and negligible in the other two dimensions. It is said to be *two-dimensional* when conduction in the third dimension is negligible and *three-dimensional* when conduction in all dimensions is significant. In heat transfer analysis, the conversion of electrical, chemical, or nuclear energy into heat (or thermal) energy is characterized as *heat generation.*

The heat conduction equation can be derived by performing an energy balance on a differential volume element. The one-dimensional heat conduction equation in rectangular, cylindrical, and spherical coordinate systems for the case of constant thermal conductivities are expressed as

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

where the property $\alpha = k/\rho C$ is the *thermal diffusivity* of the material.

The solution of a heat conduction problem depends on the conditions at the surfaces, and the mathematical expressions for the thermal conditions at the boundaries are called the *boundary conditions.* The solution of transient heat conduction problems also depends on the condition of the medium at the beginning of the heat conduction process. Such a condition, which is usually specified at time $t = 0$, is called the *initial condition,* which is a mathematical expression for the temperature distribution of the medium initially. Complete mathematical description of a heat conduction problem requires the specification of two boundary conditions for each dimension along which heat conduction is significant, and an initial condition when the problem is transient. The most common boundary conditions are the *specified temperature, specified heat flux, convection,* and *radiation* boundary conditions. A boundary surface, in general, may involve specified heat flux, convection, and radiation at the same time.

For steady one-dimensional heat transfer through a plate of thickness L, the various types of boundary conditions at the surfaces at $x = 0$ and $x = L$ can be expressed as

Specified temperature:

$$T(0) = T_1 \quad \text{and} \quad T(L) = T_2$$

where T_1 and T_2 are the specified temperatures at surfaces at $x = 0$ and $x = L$.

Specified heat flux:

$$-k\frac{dT(0)}{dx} = \dot{q}_0 \quad \text{and} \quad -k\frac{dT(L)}{dx} = \dot{q}_L$$

where $\dot{q}_0$ and $\dot{q}_L$ are the specified heat fluxes at surfaces at $x = 0$ and $x = L$.

Insulation or thermal symmetry:

$$\frac{dT(0)}{dx} = 0 \quad \text{and} \quad \frac{dT(L)}{dx} = 0$$

Convection:

$$-k\frac{dT(0)}{dx} = h_1[T_{\infty 1} - T(0)] \quad \text{and} \quad -k\frac{dT(L)}{dx} = h_2[T(L) - T_{\infty 2}]$$

where h_1 and h_2 are the convection heat transfer coefficients and $T_{\infty 1}$ and $T_{\infty 2}$ are the temperatures of the surrounding mediums on the two sides of the plate.

Radiation:

$$-k\frac{dT(0)}{dx} = \varepsilon_1\sigma[T^4_{\text{surr},1} - T(0)^4] \quad \text{and}$$

$$-k\frac{dT(L)}{dx} = \varepsilon_2\sigma[T(L)^4 - T^4_{\text{surr},2}]$$

where ε_1 and ε_2 are the emissivities of the boundary surfaces, $\sigma = 5.67 \times 10^{-8}$ W/m² · K⁴ is the Stefan–Boltzmann constant, and $T_{\text{surr},1}$ and $T_{\text{surr},2}$ are the average temperatures of the surfaces surrounding the two sides of the plate. In radiation calculations, the temperatures must be in K or R.

Interface of two bodies A *and* B *in perfect contact at* $x = x_0$*:*

$$T_A(x_0) = T_B(x_0) \quad \text{and} \quad -k_A\frac{dT_A(x_0)}{dx} = -k_B\frac{dT_B(x_0)}{dx}$$

where k_A and k_B are the thermal conductivities of the layers A and B.

Heat generation is usually expressed *per unit volume* of the medium and is denoted by $\dot{g}$, whose unit is W/m³. Under steady conditions, the surface temperature T_s of a plane wall of thickness $2L$, a cylinder of outer radius r_o, and a sphere of radius r_o in which heat is generated at a constant rate of $\dot{g}$ per unit volume in a surrounding medium at T_∞ can be expressed as

$$T_{s,\text{ plane wall}} = T_\infty + \frac{\dot{g}L}{h}$$

$$T_{s,\text{ cylinder}} = T_\infty + \frac{\dot{g}r_o}{2h}$$

$$T_{s,\text{ sphere}} = T_\infty + \frac{\dot{g}r_o}{3h}$$

where h is the convection heat transfer coefficient. The maximum temperature rise between the surface and the midsection of a medium is given by

$$\Delta T_{\text{max, plane wall}} = \frac{\dot{g}L^2}{2k}$$

$$\Delta T_{\text{max, cylinder}} = \frac{\dot{g}r_o^2}{4k}$$

$$\Delta T_{\text{max, sphere}} = \frac{\dot{g}r_o^2}{6k}$$

When the variation of thermal conductivity with temperature $k(T)$ is known, the average value of the thermal conductivity in the temperature range between T_1 and T_2 can be determined from

$$k_{\text{ave}} = \frac{\int_{T_1}^{T_2} k(T)\,dT}{T_2 - T_1}$$

Then the rate of steady heat transfer through a plane wall, cylindrical layer, or spherical layer can be expressed as

$$\dot{Q}_{\text{plane wall}} = k_{\text{ave}}A\frac{T_1 - T_2}{L} = \frac{A}{L}\int_{T_2}^{T_1} k(T)\,dT$$

$$\dot{Q}_{\text{cylinder}} = 2\pi k_{\text{ave}}L\frac{T_1 - T_2}{\ln(r_2/r_1)} = \frac{2\pi L}{\ln(r_2/r_1)}\int_{T_2}^{T_1} k(T)\,dT$$

$$\dot{Q}_{\text{sphere}} = 4\pi k_{\text{ave}}r_1 r_2\frac{T_1 - T_2}{r_2 - r_1} = \frac{4\pi r_1 r_2}{r_2 - r_1}\int_{T_2}^{T_1} k(T)\,dT$$

The variation of thermal conductivity of a material with temperature can often be approximated as a linear function and expressed as

$$k(T) = k_0(1 + \beta T)$$

where β is called the *temperature coefficient of thermal conductivity.*

SUMMARY

One-dimensional heat transfer through a simple or composite body exposed to convection from both sides to mediums at temperatures $T_{\infty 1}$ and $T_{\infty 2}$ can be expressed as

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} \qquad \text{(W)}$$

where R_{total} is the total thermal resistance between the two mediums. For a plane wall exposed to convection on both sides, the total resistance is expressed as

$$R_{\text{total}} = R_{\text{conv},1} + R_{\text{wall}} + R_{\text{conv},2} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A}$$

This relation can be extended to plane walls that consist of two or more layers by adding an additional resistance for each additional layer. The elementary thermal resistance relations can be expressed as follows:

Conduction resistance (plane wall): $R_{\text{wall}} = \dfrac{L}{kA}$

Conduction resistance (cylinder): $R_{\text{cyl}} = \dfrac{\ln(r_2/r_1)}{2\pi Lk}$

Conduction resistance (sphere): $R_{\text{sph}} = \dfrac{r_2 - r_1}{4\pi r_1 r_2 k}$

Convection resistance: $R_{\text{conv}} = \dfrac{1}{hA}$

Interface resistance: $R_{\text{interface}} = \dfrac{1}{h_c A} = \dfrac{R_c}{A}$

Radiation resistance: $R_{\text{rad}} = \dfrac{1}{h_{\text{rad}} A}$

where h_c is the thermal contact conductance, R_c is the thermal contact resistance, and the radiation heat transfer coefficient is defined as

$$h_{\text{rad}} = \varepsilon\sigma(T_s^2 + T_{\text{surr}}^2)(T_s + T_{\text{surr}})$$

Once the rate of heat transfer is available, the *temperature drop* across any layer can be determined from

$$\Delta T = \dot{Q}R$$

The thermal resistance concept can also be used to solve steady heat transfer problems involving parallel layers or combined series-parallel arrangements.

Adding insulation to a cylindrical pipe or a spherical shell will increase the rate of heat transfer if the outer radius of the insulation is less than the *critical radius of insulation*, defined as

$$r_{\text{cr, cylinder}} = \frac{k_{\text{ins}}}{h}$$

$$r_{\text{cr, sphere}} = \frac{2k_{\text{ins}}}{h}$$

The effectiveness of an insulation is often given in terms of its *R-value*, the thermal resistance of the material per unit surface area, expressed as

$$R\text{-value} = \frac{L}{k} \qquad \text{(flat insulation)}$$

where L is the thickness and k is the thermal conductivity of the material.

Finned surfaces are commonly used in practice to enhance heat transfer. Fins enhance heat transfer from a surface by exposing a larger surface area to convection. The temperature distribution along the fin for very long fins and for fins with negligible heat transfer at the fin are given by

Very long fin: $\dfrac{T(x) - T_\infty}{T_b - T_\infty} = e^{-x\sqrt{hp/kA_c}}$

Adiabatic fin tip: $\dfrac{T(x) - T_\infty}{T_b - T_\infty} = \dfrac{\cosh a(L - x)}{\cosh aL}$

where $a = \sqrt{hp/kA_c}$, p is the perimeter, and A_c is the cross sectional area of the fin. The rates of heat transfer for both cases are given to be

Very long fin: $\dot{Q}_{\text{long fin}} = -kA_c \left.\dfrac{dT}{dx}\right|_{x=0} = \sqrt{hpkA_c}\,(T_b - T_\infty)$

Adiabatic fin tip: $\dot{Q}_{\text{insulated tip}} = -kA_c \left.\dfrac{dT}{dx}\right|_{x=0} = \sqrt{hpkA_c}\,(T_b - T_\infty)\tanh aL$

Fins exposed to convection at their tips can be treated as fins with insulated tips by using the corrected length $L_c = L + A_c/p$ instead of the actual fin length.

The temperature of a fin drops along the fin, and thus the heat transfer from the fin will be less because of the decreasing temperature difference toward the fin tip. To account for the effect of this decrease in temperature on heat transfer, we define *fin efficiency* as

$$\eta_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\text{Actual heat transfer rate from the fin}}{\text{Ideal heat transfer rate from the fin if the entire fin were at base temperature}}$$

When the fin efficiency is available, the rate of heat transfer from a fin can be determined from

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}}\dot{Q}_{\text{fin, max}} = \eta_{\text{fin}} h A_{\text{fin}} (T_b - T_\infty)$$

The performance of the fins is judged on the basis of the enhancement in heat transfer relative to the no-fin case and is expressed in terms of the *fin effectiveness* ε_{fin}, defined as

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b(T_b - T_\infty)} = \frac{\text{Heat transfer rate from the fin of base area } A_b}{\text{Heat transfer rate from the surface of area } A_b}$$

Here, A_b is the cross-sectional area of the fin at the base and $\dot{Q}_{\text{no fin}}$ represents the rate of heat transfer from this area if no fins are attached to the surface. The *overall effectiveness* for a finned surface is defined as the ratio of the total heat transfer from the finned surface to the heat transfer from the same surface if there were no fins,

$$\varepsilon_{\text{fin, overall}} = \frac{\dot{Q}_{\text{total, fin}}}{\dot{Q}_{\text{total, no fin}}} = \frac{h(A_{\text{unfin}} + \eta_{\text{fin}} A_{\text{fin}})(T_b - T_\infty)}{hA_{\text{no fin}}(T_b - T_\infty)}$$

Fin efficiency and fin effectiveness are related to each other by

$$\varepsilon_{\text{fin}} = \frac{A_{\text{fin}}}{A_b}\eta_{\text{fin}}$$

Certain multidimensional heat transfer problems involve two surfaces maintained at constant temperatures T_1 and T_2. The steady rate of heat transfer between these two surfaces is expressed as

$$\dot{Q} = Sk(T_1 - T_2)$$

where S is the *conduction shape factor* that has the dimension of *length* and k is the thermal conductivity of the medium between the surfaces.

SUMMARY

In this chapter we considered the variation of temperature with time as well as position in one- or multidimensional systems. We first considered the *lumped systems* in which the temperature varies with time but remains uniform throughout the system at any time. The temperature of a lumped body of arbitrary shape of mass m, volume V, surface area A_s, density ρ, and specific heat C_p initially at a uniform temperature T_i that is exposed to convection at time $t = 0$ in a medium at temperature T_∞ with a heat transfer coefficient h is expressed as

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$$

where

$$b = \frac{hA_s}{\rho C_p V} = \frac{h}{\rho C_p L_c} \qquad \text{(1/s)}$$

is a positive quantity whose dimension is (time)$^{-1}$. This relation can be used to determine the temperature $T(t)$ of a body at time t or, alternately, the time t required for the temperature to reach a specified value $T(t)$. Once the temperature $T(t)$ at time t is available, the *rate* of convection heat transfer between the body and its environment at that time can be determined from Newton's law of cooling as

$$\dot{Q}(t) = hA_s[T(t) - T_\infty] \qquad \text{(W)}$$

The *total amount* of heat transfer between the body and the surrounding medium over the time interval $t = 0$ to t is simply the change in the energy content of the body,

$$Q = mC_p[T(t) - T_i] \qquad \text{(kJ)}$$

The amount of heat transfer reaches its upper limit when the body reaches the surrounding temperature T_∞. Therefore, the *maximum* heat transfer between the body and its surroundings is

$$Q_{max} = mC_p(T_\infty - T_i) \qquad \text{(kJ)}$$

The error involved in lumped system analysis is negligible when

$$\text{Bi} = \frac{hL_c}{k} < 0.1$$

where Bi is the *Biot number* and $L_c = V/A_s$ is the *characteristic length*.

When the lumped system analysis is not applicable, the variation of temperature with position as well as time can be determined using the *transient temperature charts* given in Figs. 4–13, 4–14, 4–15, and 4–23 for a large plane wall, a long cylinder, a sphere, and a semi-infinite medium, respectively. These charts are applicable for one-dimensional heat transfer in those geometries. Therefore, their use is limited to situations in which the body is initially at a uniform temperature, all surfaces are subjected to the same thermal conditions, and the body does not involve any heat generation. These charts can also be used to determine the total heat transfer from the body up to a specified time t.

SUMMARY

numerical *finite difference method* is based on replacing derivatives by differences, and the finite difference formulation of a heat transfer problem is obtained by selecting a sufficient number of points in the region, called the *nodal points* or nodes, and writing *energy balances* on the volume elements centered about the nodes.

For *steady* heat transfer, the *energy balance* on a volume element can be expressed in general as

$$\sum_{\text{All sides}} \dot{Q} + \dot{g}V_{\text{element}} = 0$$

whether the problem is one-, two-, or three-dimensional. For convenience in formulation, we always assume all heat transfer to be *into* the volume element from all surfaces toward the node under consideration, except for specified heat flux whose direction is already specified. The finite difference formulations for a general interior node under *steady* conditions are expressed for some geometries as follows:

One-dimensional steady conduction in a plane wall:

$$\frac{T_{m-1} - 2T_m + T_{m+1}}{(\Delta x)^2} + \frac{\dot{g}_m}{k} = 0$$

Two-dimensional steady conduction in rectangular coordinates:

$$T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} - 4T_{\text{node}} + \frac{\dot{g}_{\text{node}} l^2}{k} = 0$$

where Δx is the nodal spacing for the plane wall and $\Delta x = \Delta y = l$ is the nodal spacing for the two-dimensional case. Insulated boundaries can be viewed as mirrors in formulation, and thus the nodes on insulated boundaries can be treated as interior nodes by using mirror images.

The finite difference formulation at node 0 at the left boundary of a plane wall for steady one-dimensional heat conduction can be expressed as

$$\dot{Q}_{\text{left surface}} + kA\frac{T_1 - T_0}{\Delta x} + \dot{g}_0(A\Delta x/2) = 0$$

where $A\Delta x/2$ is the volume of the volume, $\dot{g}_0$ is the rate of heat generation per unit volume at $x = 0$, and A is the heat transfer area. The form of the first term depends on the boundary condition at $x = 0$ (convection, radiation, specified heat flux, etc.).

The finite difference formulation of heat conduction problems usually results in a system of N algebraic equations in N unknown nodal temperatures that need to be solved simultaneously. There are numerous systematic approaches available in the literature. Several widely available *equation solvers* can also be used to solve a system of equations simultaneously at the press of a button.

The finite difference formulation of *transient* heat conduction problems is based on an energy balance that also accounts for the variation of the energy content of the volume element during a time interval Δt. The heat transfer and heat generation terms are expressed at the previous time step i in the *explicit method*, and at the new time step $i + 1$ in the *implicit method*. For a general node m, the finite difference formulations are expressed as

Explicit method:

$$\sum_{\text{All sides}} \dot{Q}^i + \dot{G}^i_{\text{element}} = \rho V_{\text{element}} C\frac{T_m^{i+1} - T_m^i}{\Delta t}$$

Implicit method:

$$\sum_{\text{All sides}} \dot{Q}^{i+1} + \dot{G}^{i+1}_{\text{element}} = \rho V_{\text{element}} C\frac{T_m^{i+1} - T_m^i}{\Delta t}$$

where T_m^i and T_m^{i+1} are the temperatures of node m at times $t_i = i\Delta t$ and $t_{i+1} = (i + 1)\Delta t$, respectively, and $T_m^{i+1} - T_m^i$ represents the temperature change of the node during the time interval Δt between the time steps i and $i + 1$. The explicit and implicit formulations given here are quite general and can be used in any coordinate system regardless of heat transfer being one-, two-, or three-dimensional.

The explicit formulation of a general interior node for one- and two-dimensional heat transfer in rectangular coordinates can be expressed as

One-dimensional case:

$$T_m^{i+1} = \tau(T_{m-1}^i + T_{m+1}^i) + (1 - 2\tau)\,T_m^i + \tau\frac{\dot{g}_m^i \Delta x^2}{k}$$

Two-dimensional case:

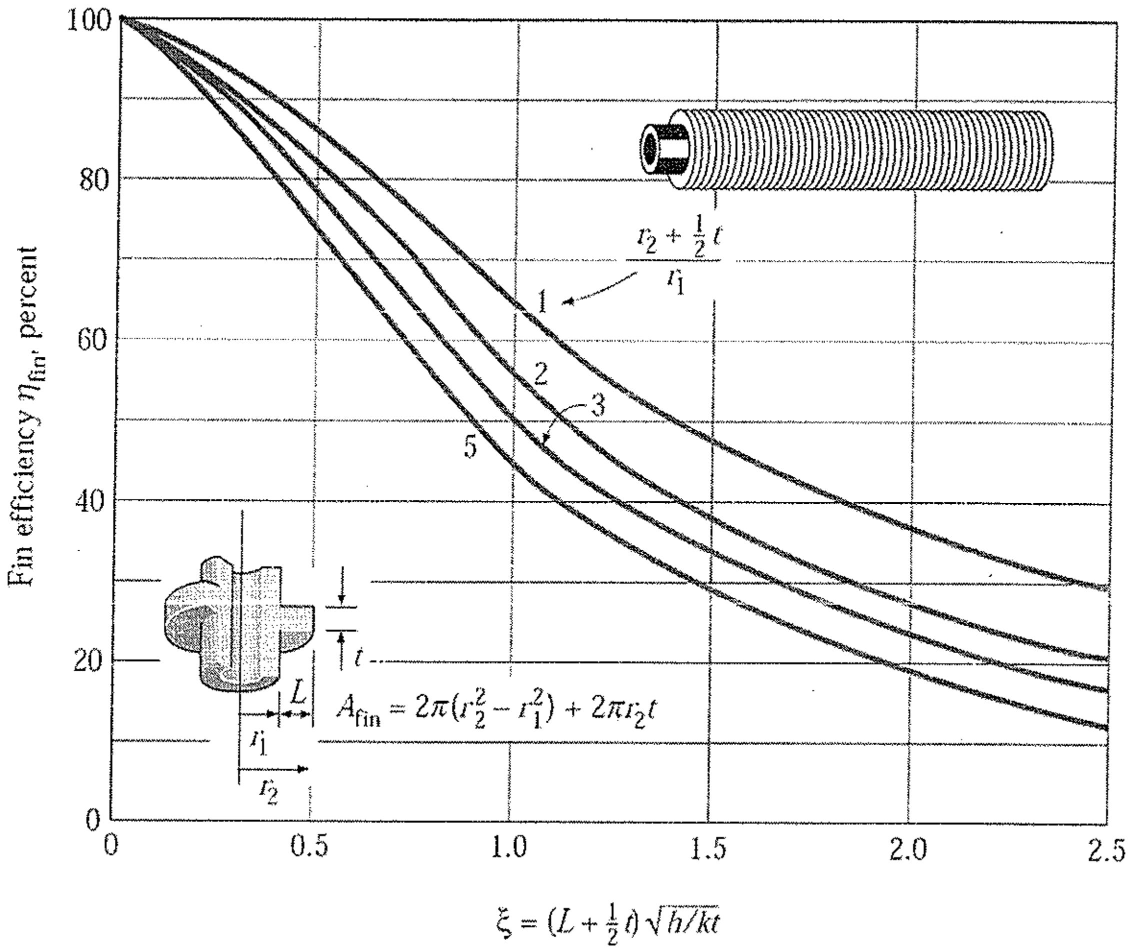
$$T_{\text{node}}^{i+1} = \tau(T_{\text{left}}^i + T_{\text{top}}^i + T_{\text{right}}^i + T_{\text{bottom}}^i) + (1 - 4\tau)\,T_{\text{node}}^i + \tau\frac{\dot{g}_{\text{node}}^i l^2}{k}$$

where

$$\tau = \frac{\alpha\Delta t}{\Delta x^2}$$

is the dimensionless *mesh Fourier number* and $\alpha = k/\rho C$ is the *thermal diffusivity* of the medium.

The implicit method is inherently stable, and any value of Δt can be used with that method as the time step. The largest value of the time step Δt in the explicit method is limited by the *stability criterion*, expressed as: *the coefficients of all T_m^i in the T_m^{i+1} expressions (called the primary coefficients) must be greater than or equal to zero for all nodes m.* The maximum value of Δt is determined by applying the stability criterion to the equation with the smallest primary coefficient since it is the



100
80
60
40
20
0
Fin efficiency η_{fin}, percent
$\frac{r_2 + \frac{1}{2}t}{r_1}$
1
2
3
5
t
L
$A_{fin} = 2\pi(r_2^2 - r_1^2) + 2\pi r_2 t$
r_1
r_2
0
0.5
1.0
1.5
2.0
2.5
$\xi = (L + \frac{1}{2}t)\sqrt{h/kt}$