INTEGRATION OF MINDLIN’S STRESS SOLUTION FOR PILED FOUNDATION APPLICATION

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Abstract: Mindlin’s stress solution is for the determination of stress distribution from point load acting within elastic homogeneous semi-infinite mass. The solution is superior to Boussinesq’s solution for piled foundation which produced the force acting within soil mass due to end bearing and skin friction. In this study, the Mindlin’s stress was integrated over the loading area of piled foundation for both normal stress (end bearing) and shear stress (skin friction). Wolfram’s Mathematica was used to obtain the integration solution for various cases of piled foundation loading. The results presented in the form of stress influence factor for further application in engineering purposes.

Key Words: Mindlin’s Stress / Elastic Homogeneous Mass / Piled Foundation / Wolfram’s Mathematica

1. INTRODUCTION

Stress distribution within soil mass from piled foundation for analysis of settlement usually carried out by using Boussinesq’s solution [1] or the approximate method to determine the stress increase at particular position. In this paper, the Mindlin’s stress [2] was integrated over the loading area of piled foundation. Because piled foundation transfer load to the soil mass by end bearing stress at pile tip and skin friction along the side area as shown in Fig. 1. The Mindlin’s solution for load acting within the ground is superior to Boussinesq’s solution for application in engineering purposes.

2. THEORY

For a point load acting within soil mass normal to plane, the stresses at point \((r, z)\) can be determined as shown in Fig. 2.

For a horizontal point load acting within soil mass parallel to plane, which resembled the skin friction plane of piled foundation, the stress at point \((r, z)\) can be determined as shown in Fig. 3.
The solution is shown in following equation:

\[
\Delta \sigma_x = \frac{Qx}{8\pi(1-\nu)} \left\{ \frac{(1-2\nu)(5-4\nu)}{R_1^3} \right. \\
- \frac{3x^2}{R_1^3} \left( 3(3-4\nu)x^2 \right) R_2^3 \\
- \frac{4(1-\nu)(1-2\nu)}{R_3(R_2+z+c)} \left[ 3 - \frac{x^2(3R_2+z+c)}{R_2^2} \right] \\
+ \frac{6c}{R_2^3} \left[ 3c - (3-2\nu)(z+c) + \frac{5x^2z}{R_2^2} \right] \right\}
\]  \( (2) \)

3. INTEGRATION OF MINDLIN’S STRESSES

For the uniformly distributed load acting normal to the plane \( b \times l \), which resemble the end bearing of the piled foundation, the single point load normal to loading area can be integrated for the stress at point \((s, t, z)\) as shown in Fig. 4.

From equation (1) the stress condition such as:

\[
\Delta \sigma_z = \frac{Q}{8\pi(1-\nu)} \cdot K_z \\
\]

and integrated by:

\[
\Delta \sigma_z = \frac{Q}{8\pi(1-\nu)} \int K_z \cdot dA \\
\]

The force acting parallel to the plane, can also integrated by using Mindlin’s solution for horizontal force acting within soil mass in the vertical direction, which will resemble the skin friction along side of the foundation. The solution for one side of the pile can be expressed as shown in Fig. 5.

From equation (2) for horizontal stress such as:

\[
\Delta \sigma_x = -\frac{Qx}{8\pi(1-\nu)} \cdot K_x \\
\]

For the vertical direction condition such as:

\[
\Delta \sigma_z = \frac{Q(z-h)}{8\pi(1-\nu)} \cdot K_z \\
\]
and integrated by:
\[
\Delta \sigma_z = \frac{q}{8\pi(1-\nu)} \int K_z(z-h) \cdot dA
\]  
(17)
\[
\Delta \sigma_z = \frac{q}{8\pi(1-\nu)} \int\left[ K_z(z-h) \right] dhdy
\]  
(18)
The equation result in the form:
\[
\Delta \sigma_{z,f} = \frac{Q}{L_p \times 2(h+l)} \cdot K_f
\]  
(19)
where \( K_f = f(s,t,z,b,l,L_p,\nu) \).

For all four sides of the piled foundation, the skin friction stress is the sum of stress at proportional positions of all four sides expresses as shown in Fig. 6.

![Fig. 6. Side of the piled foundation](image)

The result is shown in following equation:
\[
\Delta \sigma_{z,f} = \frac{Q}{L_p \times 2(h+l)} (K_{f1} + K_{f2} + K_{f3} + K_{f4})
\]  
(20)

When combine the stresses at a point due to normal stress and shear stress in the ratio similar to the application of total load transfer from pile \( Q_u \) as end bearing \( Q_e \) and skin friction \( Q_f \), in such a manner that:
\[
e = \frac{Q_e}{Q_u}, \quad f = \frac{Q_f}{Q_u}
\]

We will get:
\[
\Delta \sigma_z = e \cdot \Delta \sigma_{z,e} + f \cdot \Delta \sigma_{z,f}
\]  
(21)

4. RESULTS

The results as the influence factor at points for end bearing and skin friction stresses are shown in Fig. 7.

![Fig. 7. End bearing and skin friction stresses influence](image)

For example, a piled foundation carried total \( Q_u \) of 100 t composed of end bearing \( Q_e \) of 25 t and skin friction \( Q_f \) of 75 t, as shown in Fig. 8. The stress at \( 2s/b = 1.5 \) and \( (z-L_p)/b = 0.5 \)

![Fig. 8. Pile foundation loading for example](image)

Can be found as:
\[
\Delta \sigma_z = 0.18 \frac{Q_e}{Area} + 0.42 \frac{Q_f}{Area}
\]
\[
\Delta \sigma_z = 0.18 \frac{25}{1 \times 1} + 0.42 \frac{75}{1 \times 2(1+1)}
\]
\[
\Delta \sigma_z = 7.875 \text{ t/Unit of area}
\]

It was found that the results are more reasonable than the assumed Boussinesq’s or approximate method for load application below ground surface [3].

5. CONCLUSION

The Mindlin’s stress for point load application within soil mass was integrated over the loading area in this study using Wolfram’s Mathematica. The solution is applicable for piled foundation loading. The stress influence factor for end bearing and skin friction were determinate and plotted to ease the calculation of stress distribution within soil mass.

REFERENCES

